Cranial Sutures as the Fractal Coastlines

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The cranial bones are usually joined by complex osseous projections, which precisely interlock. The image of such connections between two cranial bones resembles outline of the coastline with many bays. The contour of these lines may vary from slightly convoluted lines to complex patterns that resemble coastlines with many peninsulas and bays [4]. When a selected segment of cranial suture is analyzed, one may perceive similar property of curviness. This character is typical for fractal structures and it is termed as self-similarity. However, the cranial sutures are not typical fractals and self-similar properties are limited to 2-3 orders of scaling but they yield fractal dimension, which differs with kinds of suture-lines [5]. In this respect fractal dimension becomes a useful index for determining sutural morphology in terms of its complexity.

The cranial sutures can be modeled as lines that have been deformed in a particular way, demonstrating interfingering and lateral excursions [3]. Possible formation of cranial suture contour could be explained by the random midpoint displacement, which is widely used to depict fractal coastlines or fractal landscapes [6, 9]. Here are the rules of this algorithm and its initial steps are presented in the *figure 1*.

1. Make a straight line
2. Grab the middle of the line and move to one of the side (eg. right or left) by a random amount.
3. Then take one of the two new segments and drag the middle point toward the right or left side
4. Recursively repeat this process with smaller segments
5. Stop the process when the individual segments are too short to be worth to be splitting.

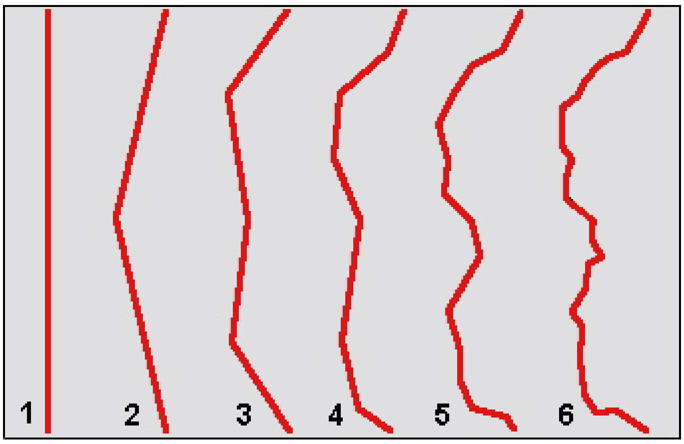


Figure 1 Six steps of the random midpoint displacement method applied to the line

The method starts with a simple line and recursively adds random details, which number of elements is equal 2^n, where n is the step number. Each step produces twice as many segments of the line with random displacement as the step before. A brief mathematical description of the random midpoint displacement algorithm can be formulated in the following way:

Assume that values $X(0) = 0$ and $X(1)$ are given. $X(1)$ is obtained as a Gausian random number. The next step refers to the partition the interval [0,1] into two subintervals: [0, 1/2], [1/2, 1], and the $X(1/2)$ is defined as the mean of $X(0)$ + i(1) plus a displacement D as the Gaussian. The big subdivision gets a big displacement, while the smaller subdivisions get smaller displacement, what makes the line fractally. The initial three steps of the midpoint displacement algorithm are presented as:

$$X(1/2) = [X(0) + X(1)]/2 + D_1$$
$$X(1/4) = [X(0) + X(1/2)]/2 + D_2$$
$$X(3/4) = [X(1/2) + X(1)]/2 + D_3$$

This process is continued with displacements D_n having variance Δ_n^2. In the result, the baseline is displaced by the fractal line, which looks like a silhouette of the coastline [8].

The entire process of the random midpoint displacement is based on randomly generated numbers and usually controlled by three variables: the number of iterations, roughness and the number of initial points. Here are the definitions of these variables [1, 10]:

- Random Number – are generated by the computer uses a function called "random number generator." The random number generator follows a normal random distribution function, with mean $\mu = 0$ and variance $\sigma^2 = 1$. A different distribution function, or a different mean or variance would generate different random numbers, and thus different coastlines.
- Roughness – is the factor by which the perturbations are reduced on each iteration. Higher values result in a smoother surface while lower values result in a rougher surface. Roughness (R) is also related to scale (S) by the exponential relationship: $S = (1/2)^R$. Thus, a larger scale means a smaller roughness value, and a rougher landscape.
- Initial points – points where the coastline is broken during its formation. They are specified to provide some degree of control over the appearance of the coastline. Usually, the number of initial points may very from 2 to 5.
- Number of iterations – indicate how many times the process of the midpoint displacement is repeated. At n iterations, there will be 2^n segments of the line. The larger number of iterations means the smaller parts on the initial line and it results in the increase of the polyline segments.

The procedure of random midpoint displacement can be performed by software and the results are visible on the computer screen. The example of such software can be found in the web [2, 6,10].

The appearance of the generated coastlines resembles the outlines of the cranial bones, which are joined by the sutures. In other words, the algorithm of random midpoint displacement becomes a model of sutural pattern formation (compare *figure 2*, *figure 3* and *figure 4*).

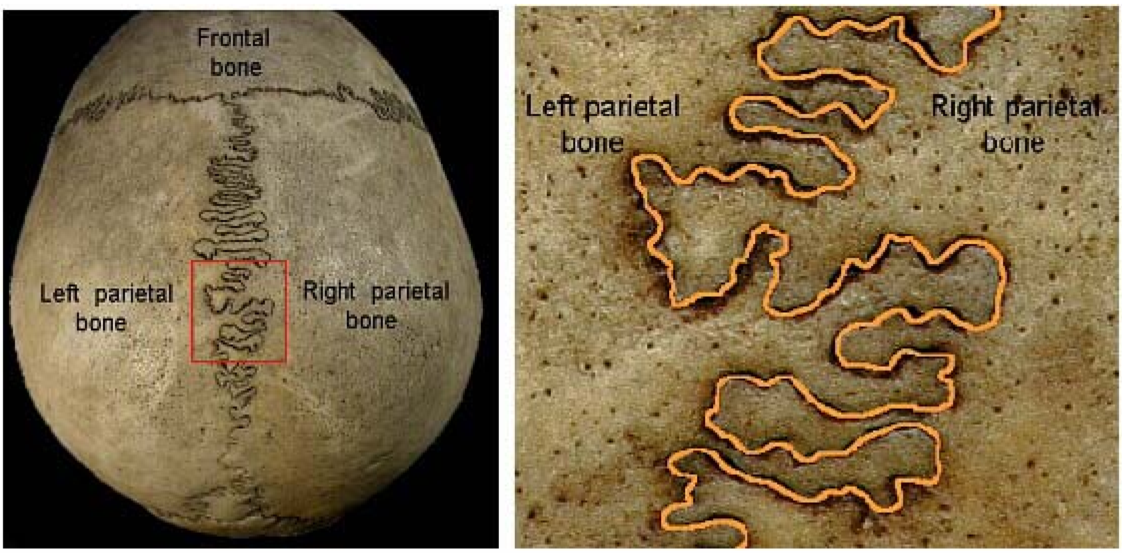


Figure 2 Image of the cranial vault with demarcated sagittal suture that joins two parietal bones. Red rectangle marks the region of interest (ROI), which is magnified in the opposite picture. The orange line represents suture contour or the border between two parietal bones joined by the suture.

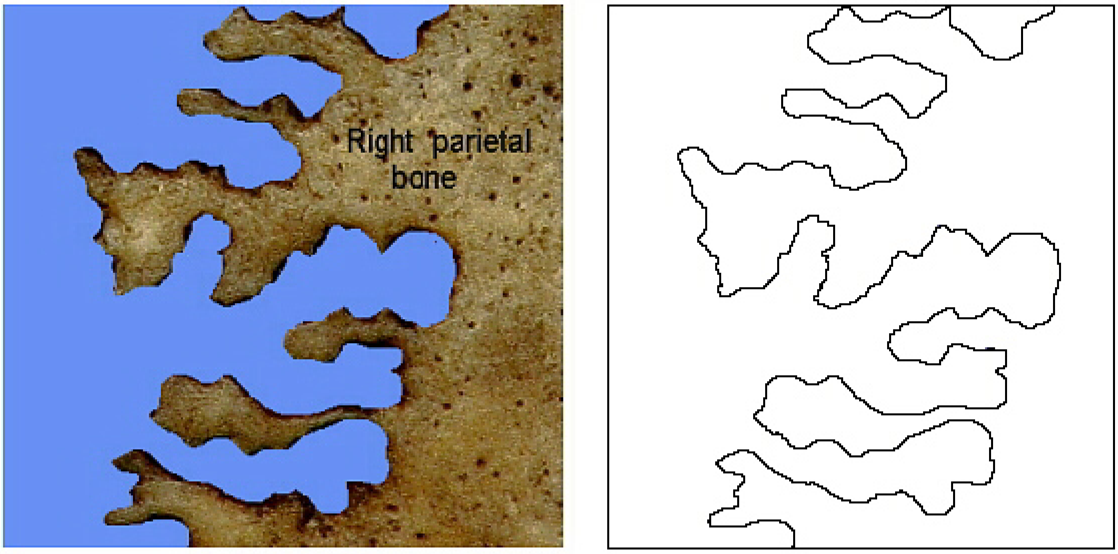
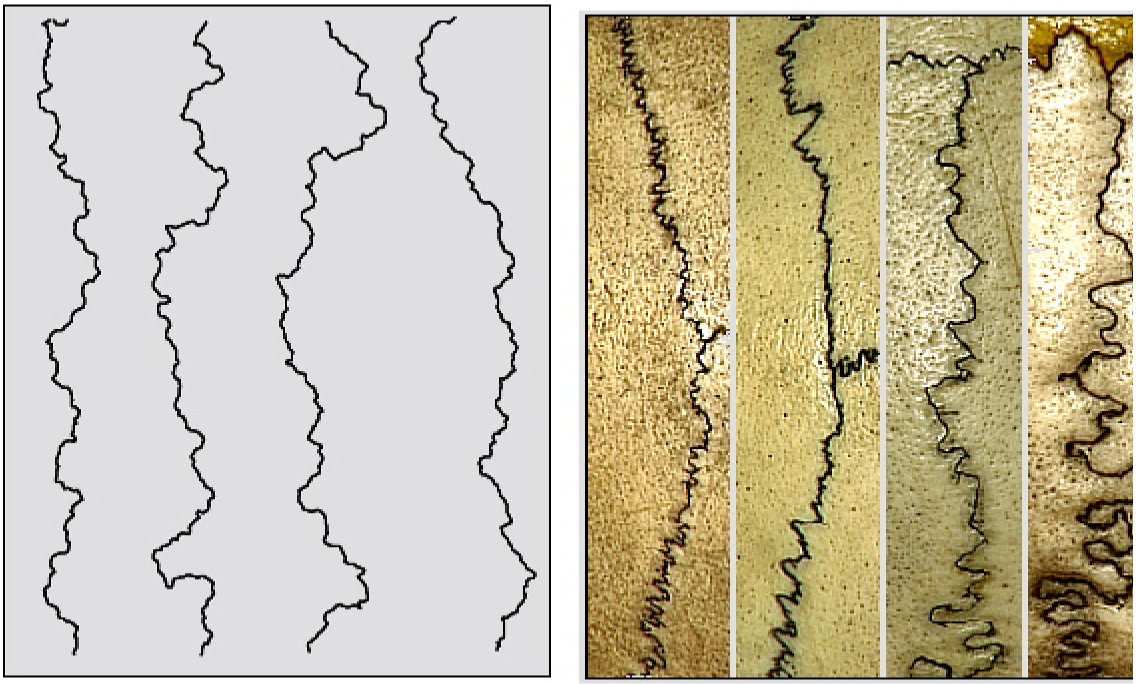


Figure 3. The edge of the right parietal bone within the ROI, (left parietal bone is removed). The isolated one pixel line as a coastline represents pattern of cranial suture



a *b*

Figure 4. Examples of computer generated coastlines (a) using random midpoint displacement algorithm and natural patterns of cranial sutures (b)

Computer generated curves were compared to the curves of real cranial sutures by means of fractal dimension, which was measured as the box-counting dimension with implementation the HarFA software [7]. The box-counting dimension (D_F) is defined as:

$$D_F = \lim_{\varepsilon \to 0} \frac{\log N(\varepsilon)}{\log(1/\varepsilon)},$$

where: $N(\varepsilon)$ is the number of boxes of side-length ε needed to cover the analysed object.

The box-counting dimensions of the computer-generated coastlines were compared to the fractal dimensions of the natural cranial sutures. As the result we got similar values of fractal dimensions ranging from 1.10 – 1.40 and moreover these values remain consistent with the fractal dimensions calculated by other researchers [3, 5, 11]. This indicates those computer-generated patterns and cranial sutures show similar complexity and because of their appearance and character they were regarded to belong to the same class of geometrical objects termed as fractal structures.

The proposed coastline model of the cranial suture is a simplification as it represents only the outline of the edge of the cranial bone and not entire surface, which faces with the opposing bone. However, this algorithm can be extended into 2D structures and then it will become more appropriate for modeling surfaces of the cranial bones, which contact within the suture. Moreover, such mechanism of structure formation, in the case of cranial suture might be possible in certain range of scale and it does not have to be universal for all sutures. Certainly, the suture morphogenesis is highly complex process, which is dictated by various factors and their interactions are not based on singular algorithm. The midpoint displacement method produces natural-like object patterns, which appearance corresponds to cranial suture morphology. However, sutures, which are obtained with random midpoint displacement algorithm sometimes, are not ideal representations of the real cranial suture because of produced unnatural features. Therefore their appearance depends significantly on relationship between values of three parameters: roughness, the number of initial points and the number of iterations. Nevertheless, this method models geometrical construction, which might be attributed to the cranial sutures. Such a model may help to understand mechanisms of changes in the bone edges configuration when they form suture during skull development. This algorithm shows also clearly the idea around implication of fractal geometry not only to describe contours of the cranial sutures but also to explain hypothetical mechanism that may be engaged in suture morphogenesis.

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