Fractality in the *Nautilus Pompilius* Shell

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The complexity of the *Nautilus Pompilius* shell is analysed in terms of its fractal dimension and its equiangular spiral form. Our findings assert that the shell is fractal from its birth and that its growth is dictated by a self-similar criterion (we obtain the fractal dimension of the shell as a function of time).

1. Introduction

Fractal analysis is being applied with increasing frequency to living organisms, trying to explain some of the complex forms found in nature. An astonishing example reveals that *Ammonites* continuously increased their complexity up to the point in which they became extinct [1]. It is our purpose to study in this paper the amazing complexity of a close relative of the *Ammonites*, the *Nautilus pompilius*.

This pelagic species is a native of the western Indopacific ocean (30° N lat. to 30° S lat. and 90° to 185° W long. [2]), and usually lives at a depth that varies from 50 to 480 meters (temperature ranges from 24 to 8 Celsius degrees).

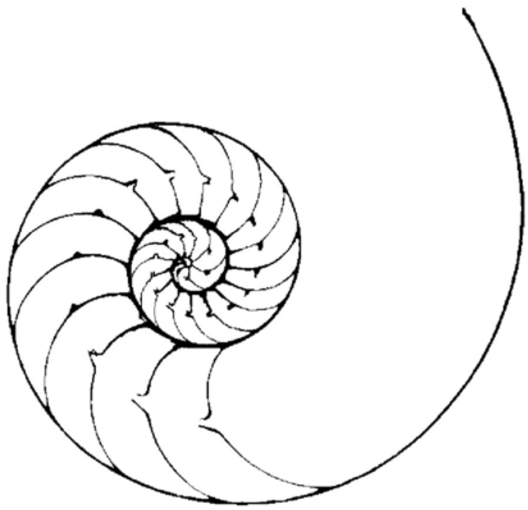

Fig. 1 Black and white image of a transversal cut of a Nautilus pompilius *shell.*

The shell is mother-of-pearl lined and pressure resistant (it implodes at approximately 800 m); its hardness has been the basis of various ornamental handicrafts [3]. But the most striking characteristic of this thin, two layered, and spirally coiled shell is its internal subdivision in a series of successive chambers (phragmocone), starting from the very moment of hatching when there are already seven chambers present in the shell. As the cephalopod grows and requires more space, it creates a new chamber by sealing the space behind it with a calcareous septum and moves to live at the open, bigger end of the shell. The rate at which a new chamber is created varies, at the beginning it seems to take longer for the mollusc to seal the 8th chamber but later on, the process takes from 43 to 77 days per chamber [4] and lasts up to the completion of approximately 39 sealed chambers [5] plus the open space where the mollusc lives [6]; these changes in the growth rate are easily understood in terms of the food availability and other environmental variables. The sealing of the chambers however, is not complete, there is a small duct in the center of each wall, called siphuncle, that allows the living fossil to keep control of the pressure inside every previous chamber and thus to regulate its buoyancy [7], [8]; the heyday of the nautiluses is estimated to be around 500 million years ago.

A transversal cut of the shell, *Fig. 1*, shows a perplexing spiral geometry, not found in any other natural object; this is a black & white image where the borders have been prepared to facilitate the box-counting analysis. The hemishell is 96.1×106.2 mm and 32.2 mm wide; the number of chambers is 30. Most amazing is the fact that its growth appears to be self-similar, and thus for the shell to possess a fractal dimension. We now proceed to confirm that this is indeed so.

2. Method

The digital image in Fig. 1 was obtained by placing half of the shell directly on a scanner bed; the cutting 2 was performed going through half of the shell as accurately as possible. All measurements are performed on the digital images, in pixel units, and the conversion factor is given by the scanner resolution (72 pixels per inch). The borders of the edges in the hemishell were previously tinted to gain contrast and improve definition, and thus, making the contour threshold treatment unnecessary.

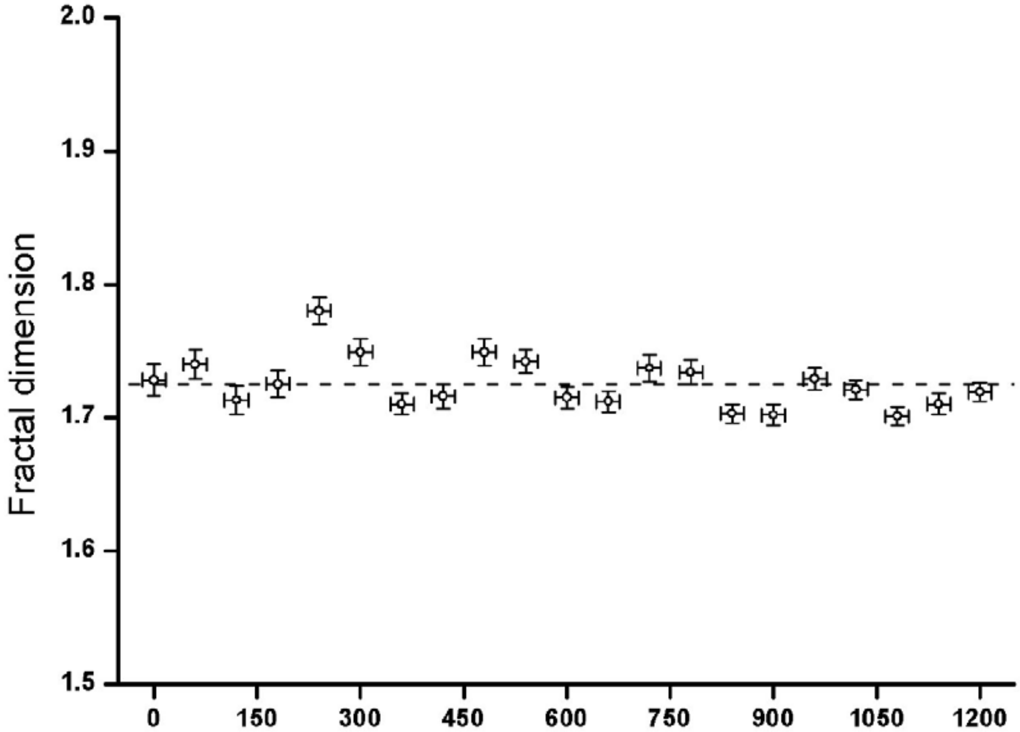

Fig. 2 Fractal dimension of the shell as a function of time, the age is measured in days after hatching, starting with the 8^{th} chamber

The box counting method (with HarFA) is applied to the original image and the fractal dimension of the whole shell is obtained via a linear fit to the data [9]. In order to test the observed self-similarity, we analyse the fractal dimension of smaller fragments of the image, that is, if we check that its complex structure is the same regardless of the scale used to measure it. To accomplish this test, we proceeded as follows; once the box-counting method had been applied to the whole, bigger image, the last chamber was digitally eliminated from the initial image and the method reapplied to the new image after adjusting the maximum possible size to the new, smaller image size. This procedure was repeated up to the point in which there were only the original seven chambers in the shell. We have also used an average value for the time required for the construction of a new chamber in order to obtain the fractal dimension of the shell as a function of time, *Fig. 2*, this average value is 60 ± 17 days per chamber.

3. Results

The fractal (box-counting) dimension of the original *Nautilus* shell shown in *Fig. 1* is 1.635 ± 0.006; the average of the self-similar fractal dimension of the shell (*Fig. 2*) is 1.730 ± 0.019; this is an average over the life of the particular *Nautilus* and clearly depends on the accuracy of the available data on the shell growth. The lower value for fractal dimension of the original shell with respect to the average, is due to the fact that the shell extension where the mollusc lives is included in the original image (*Fig. 1*)

4. Conclusions

In the previous analysis, we have shown that the shell of the *Nautilus pompilius* that we have analysed, possesses a fractal dimension, that its value is 1.635 ± 0.006 (1.730 ± 0.019 on average), and that it does not depend on the number of chambers (or, equivalently, the age) used to calculate it. This establishes the self-similar structure of the shell at any scale/time, and how its growth follows the same self-similar criterion.

5. References

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