Fractal geometry for the characterisation of urban-related states: Greater Montreal Case

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This paper summarises a brief experimentation of fractal geometry applied to the characterisation of urban-related states in the Greater Montreal Area (GMA). Research in the field of travel behaviour and urban modelling can be classified according to four basic axes of concern: the measure of urban mobility, the assessment of urban land use, the monitoring of demographic and economic forces and the appraisal of the role of the transportation networks. Fractal measures, used in conjunction with other measures, could lead to renewed description of several issues related to those axes of concern. As an introduction, methods for computing fractal measures were experimented in order to assist in the description of the land and transportation network coverage, as well as in the study of the dynamics of settlements over the area.

1. Methods for computing fractal dimensions

Two methods were used in order to estimate the fractal dimension of the urban states: the box-counting method as implemented in *Harfa* and the mass-radius method computed within a GIS environment. Results from both methods are presented.

1.1. Box-counting method

This method computes the number of cells required to entirely cover an object, with grids of cells of varying size. Practically, this is performed by superimposing regular grids over an object and by counting the number of occupied cells. The logarithm of N(r), the number of occupied cells, versus the logarithm of 1/r, where r is the size of one cell, gives a line whose gradient corresponds to the box dimension. A refinement of this method is implemented in *Harfa* where a distinction between completely occupied cells and partially occupied cells is introduced. This allows the computation of several box dimensions by plotting the logarithm of combination of cells: completely occupied, completely non-occupied, partially-occupied.

Box-counting method relies on digitized representations of the objects of interest and will be affected by their resolution. It is also sensible to the orientation of the grid as well as to its initial placement. In addition, treatment of phenomenon involving intensities (number of dwellings per enumeration areas) will require special processing such as the use of graduated symbols arbitrarily specified.

1.2. Mass-radius method

The mass dimension defines the relationship between the area located within a certain radius and the size of this radius (or box). This is performed for various radiuses as well as from various points of origin. The mass dimension can be estimated from the log-log plot of the area as a function of the radius.

In our case, the center of mass of the territory is computed and serves as origin point. The area located within a growing radius (1 to 62 km) is estimated using GIS capabilities. Moreover, urban phenomena such as population dispersion or transit share are often examined as a function of distance to CBD (Central Business District). In this view, mass dimension is also computed in reference to this point.

2. Demonstration

Fractal geometry will be experimented to characterise the morphology of the Greater Montreal Area (5 390 sq. km.) and of the spatial extent of the transportation network (1 927 sq. km.). Data from the 1996 Canadian census will also be used to illustrate the dynamics of settlements construction over the territory. Those data are disseminated at the enumeration area (EA) level (app. 250 households/EA and more than 4 500 EA in the GMA) and construction information is available for six periods: before 1946, 1946-1960, 1961-1970, 1971-1980, 1981-1990 and 1991-1996.

2.1. The Greater Montreal Area

Box counting dimension was computed over a black and white representation of the area (948 X 891 pixels), hence no threshold operation was necessary. Dimension obtained while considering both completely and partially occupied cells is 1.8583.

Mass dimension was computed using both GIS and spreadsheet functions. Results from considered origins, mass centre and central business district, are presented in *Figure 1*.

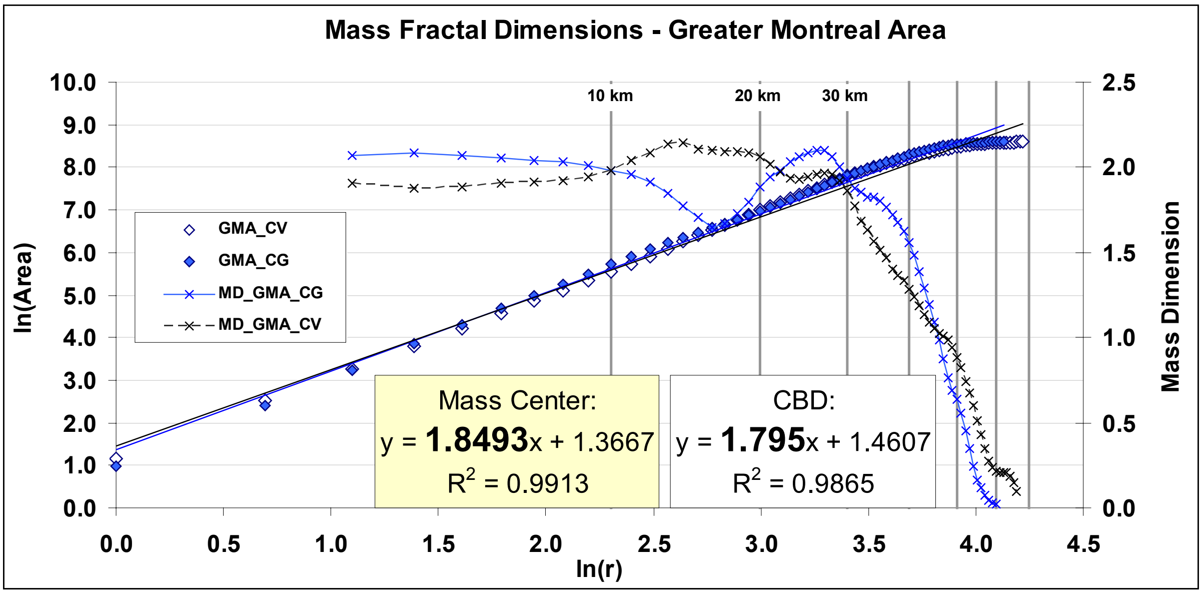

Figure 1 - Mass dimensions for Greater Montreal Area - from mass centre and central business district

The mass dimension computed from the Mass Centre, 1.8493, approaches the one measured with the box-counting method. These plot however reveals a non-linear relation between the area and the radius, especially over 30 kilometres. The fluctuation of the mass dimension can be appreciated with the rough estimation of the slope with successive data pairs; it severely drops over 30 kilometres, the radius of gyration.

The two following figures illustrate the superimposition of radiuses from mass centre and CBD.

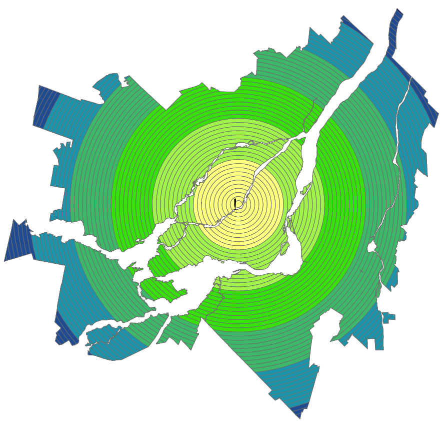

Figure 2 - 1 kilometre radius from Mass Center

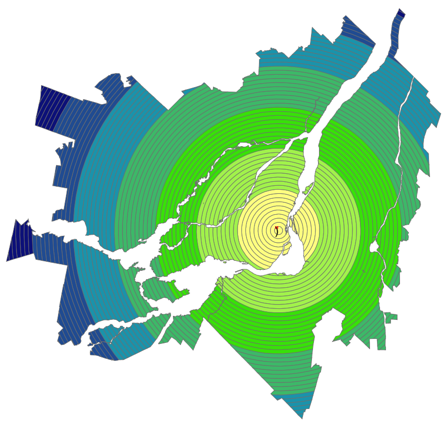

Figure 3 - 1 kilometre radius from CBD

2.2. The transportation network

A similar approach was used in order to estimate the fractal dimension of the surface covered by the transportation network. A 100 metres buffer applied over the entire network approximates this surface. The dimension and resolution of the digitised representation were maintained identical to the one used for GMA estimations (and will be preserved for the study of settlements construction).

The box-counting dimension obtained while considering both completely and partially occupied cells is 1.7392, which appears consistent with the previous results.

The computation of mass dimensions with the selected origins is synthesised in *Figure 4*. Again, the plot reveals a non-linear relation between the area and the radius that is less adequately modelled by a linear regression. The curve representing the fluctuation of the mass dimension is declining, affected by the dedensification of the network towards the suburbs.

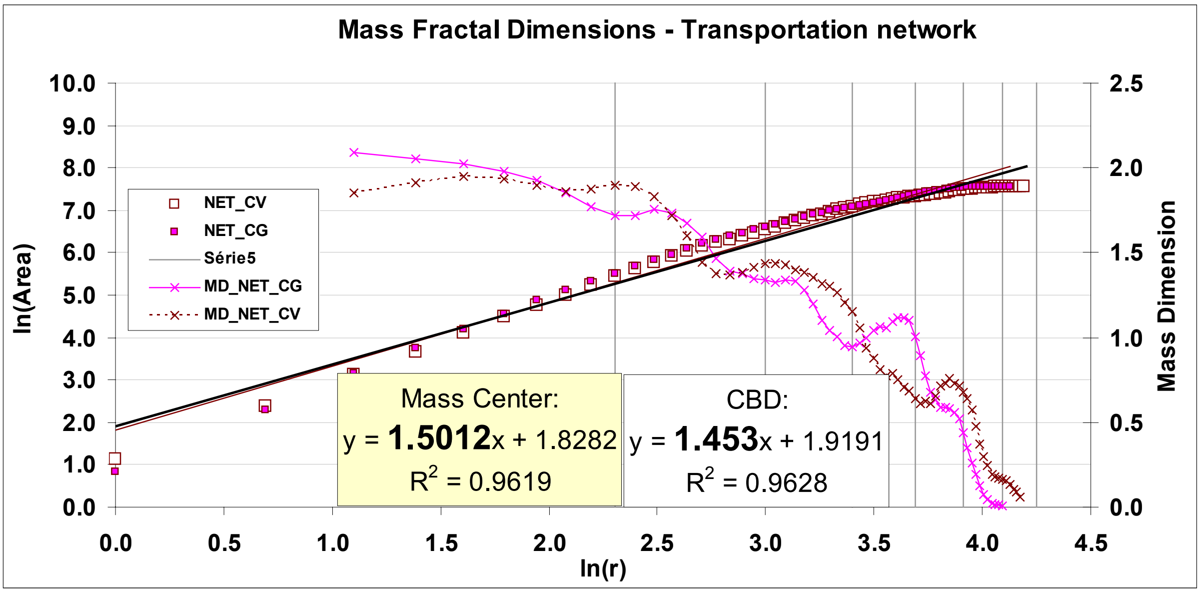

Figure 4 – Mass dimensions for the Transportation Network - from mass center and central business district

2.3. The dynamics of settlements patterns

Our final experimentation deals with dwellings' construction data from the 1996 Canadian census. Information are available for six periods of construction (number of dwellings constructed in every period) and are disseminated at the enumeration area (EA) level, one EA containing data for approximately 250 households. Since data are aggregated and spatially located according to centroids (app. 4500 to cover the GMA), it was decided to use graduated symbols for the consideration of intensities, that is the number of dwellings constructed at a specified period at every location. This processing, arbitrary for the moment, allowed the estimation of box-counting dimensions at six different stages of residential development. The results are summarised below.

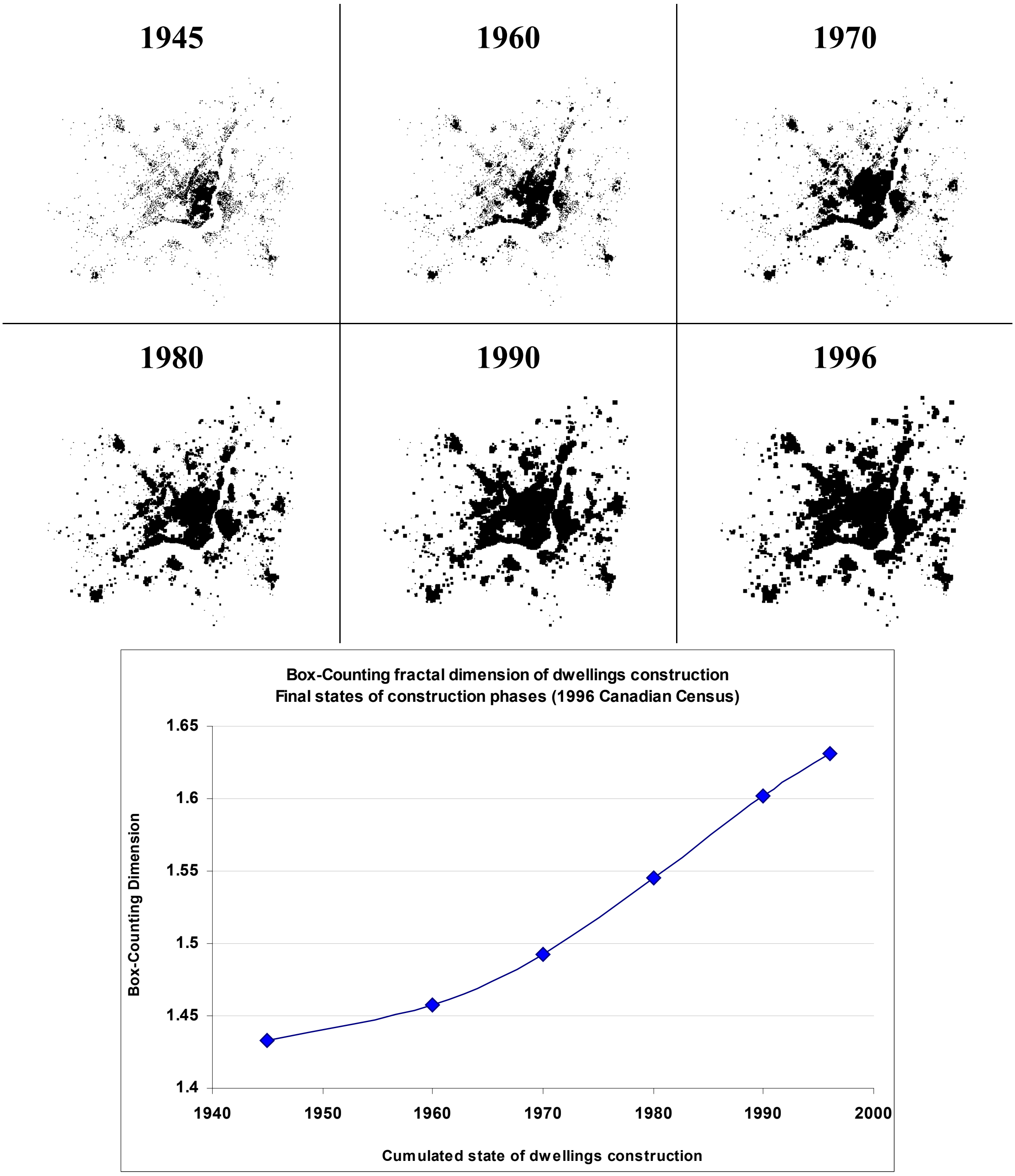

Figure 5 - Evolution of the box-counting dimension of residential construction over the GMA

3. Conclusion

At this stage, only computation feasibility was experimented over urban-related data. Interpretation of the fractal dimensions as well as demonstration of relevance/irrelevance for the modelling of transport modelling issues needs further experimentation.

4. Literature

Fractal geometry:

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