Fractal Dimension Analysis of Hollow-Cone Darkfield Images

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Abstract. The fractal dimension is used as a guide to setting the threshold for converting darkfield images into binary images. This can be used to support the existence of a preferred orientation in a thin film (Ag), or to get quantitative information about the phase content of multi-phase films (Ni-Ni_3P).

Summary.

Setting the threshold when converting a darkfield image to a binary one, to get quantitative information about orientation or phase content, can be tricky. It depends on the operators intuition as to when he or she feels the binary image is a good representation of the micrograph, and when comparing a series of images, can often lead to erroneous trends.

Box counting is overlaying a grid of boxes of side s and counting the number of boxes that contain part of the image $N(s)$. The fractal dimension D is defined as

$$N(s) = s^{-D}. \tag{1}$$

D governs the rate at which N changes with s.

One can estimate by visual inspection of an image some bounds on the expected form of the *log (N(s)) vs. log (1/s)* curve from a box counting routine. The threshold may then be set to meet those expectations. The operator may then compare the resulting binary image to the darkfield one, and if it’s a good representation visually, then there is a supporting mathematical basis for it.

There are several regimes for the fractal dimension D. Consider an image that is a series of parallel lines of length L and spacing d, and width of line w. $L >> d >> w$. As the box size s falls, the dimensionality changes,

1. $D = 0$ for $s>>L$ (2)

2. $D \rightarrow 2$ for $d < s < L$ (3)

3. $D \rightarrow 1$ for $w < s < d$ ($D \sim 0$ if these are particles of diameter w) (4)

4. $D \rightarrow 2$ for $s < w$. (5)

5. $D = 2$ for $s<<w$ (6)

That example illustrates that the D computed from the slope of *log (N(s)) vs. log (1/s)* depends on magnification, density, feature size, and the size of the "measuring stick".

With this kind of visualization tool, consider the hollow cone darkfield images. They are an ideal case, as they are taken at the same magnification, are about the same size, and have similar illumination conditions. In the upper right hand corner of the 220 darkfield collage is a feature that looks like a handprint. The "palm" part of the print is 45 pixels wide. Our boxes have $s=2,3\ldots17$ pixels per side.

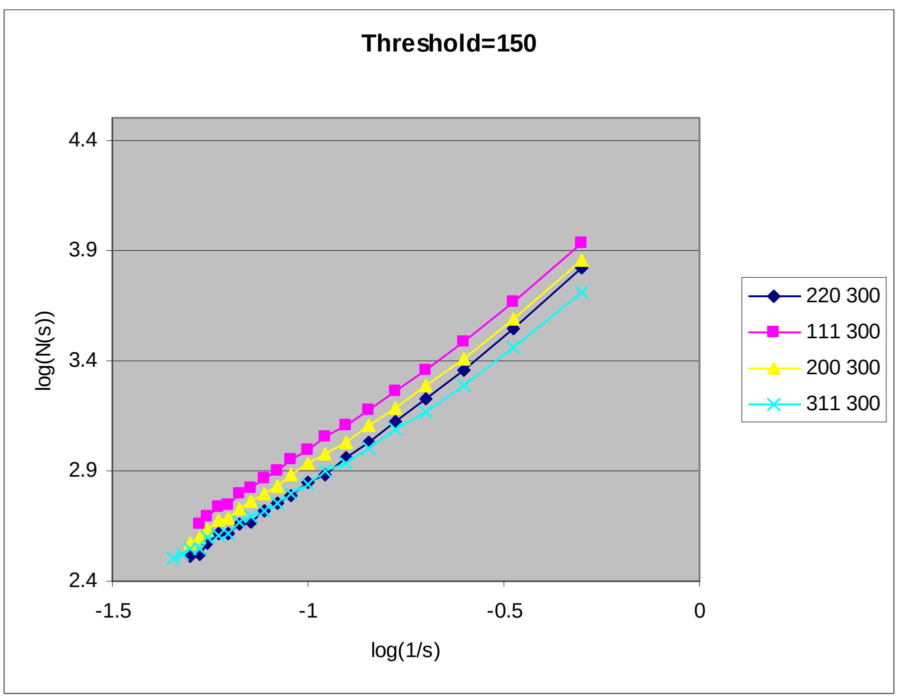
I assume that if an area shows *any contrast* and is not completely black, its coming from grains oriented perpendicular to the *hkl* of whatever ring is being examined. Note all the grains in those images are nearly space filling and connected so *D>1*. Most of the time $s \leq w$ where *w* is the feature size so $D \rightarrow 2$ as $s << w$. Also $s \leq d$ where *d* is the spacing between feature edges. As *s* decreases, the number of boxes of side *s*, *N(s)*, that contain a portion of the image should increase almost as $A_{diffract}/s^2$, but slightly slower since some boxes won't be counted due to the spacing between unconnected grains, so *D* must be slightly less than *2*. In addition, *s* is in pixels and does not range over many orders, so the slope *D* of *log (N(s)) vs. log (1/s)* will stay less than *2* and not vary too much.

This reasoning suggests setting the threshold until *log (N(s)) vs. log (1/s)* has almost the same slope for small boxes as it does for big boxes, and the slope *D* should be a bit less than two, but not too far away.

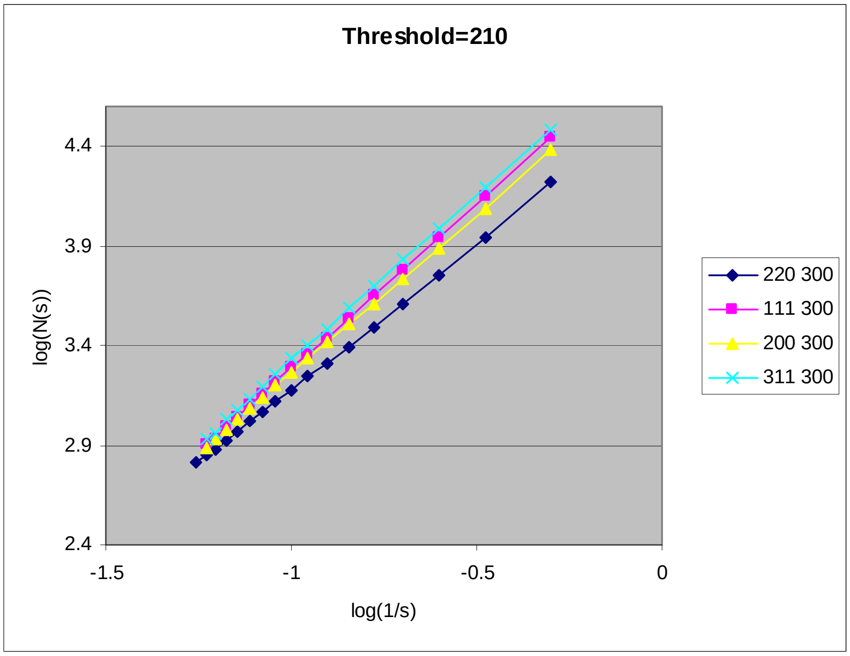
Fractal dimensions were computed with threshold settings for the 220, 111, 200, 311 hollow cone darkfield images for an Ag film on Si_3N_4, Table 1. By *threshold = 210* the small and large *s* fractal dimensions agreed more closely.

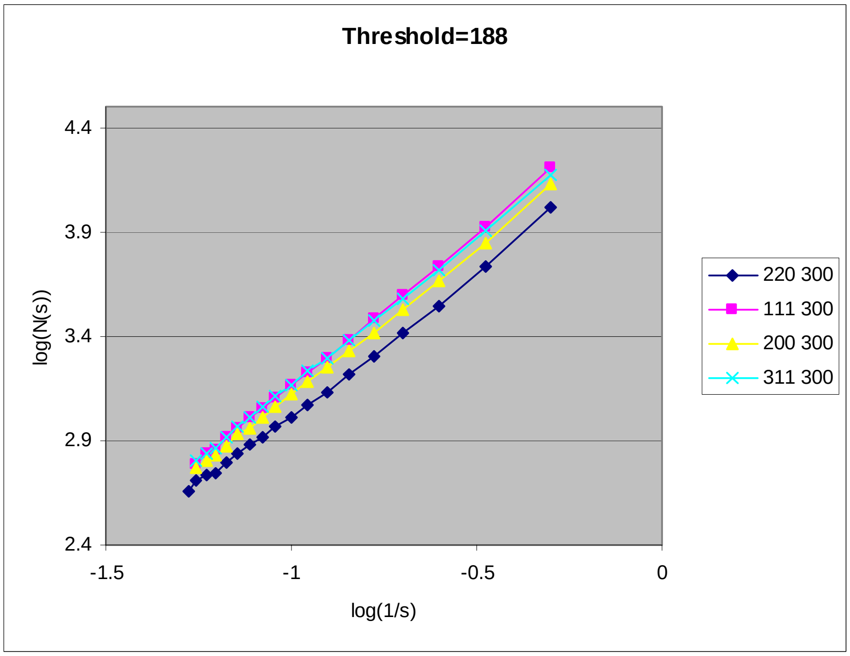
Table 1. Fractal dimensions with threshold setting.

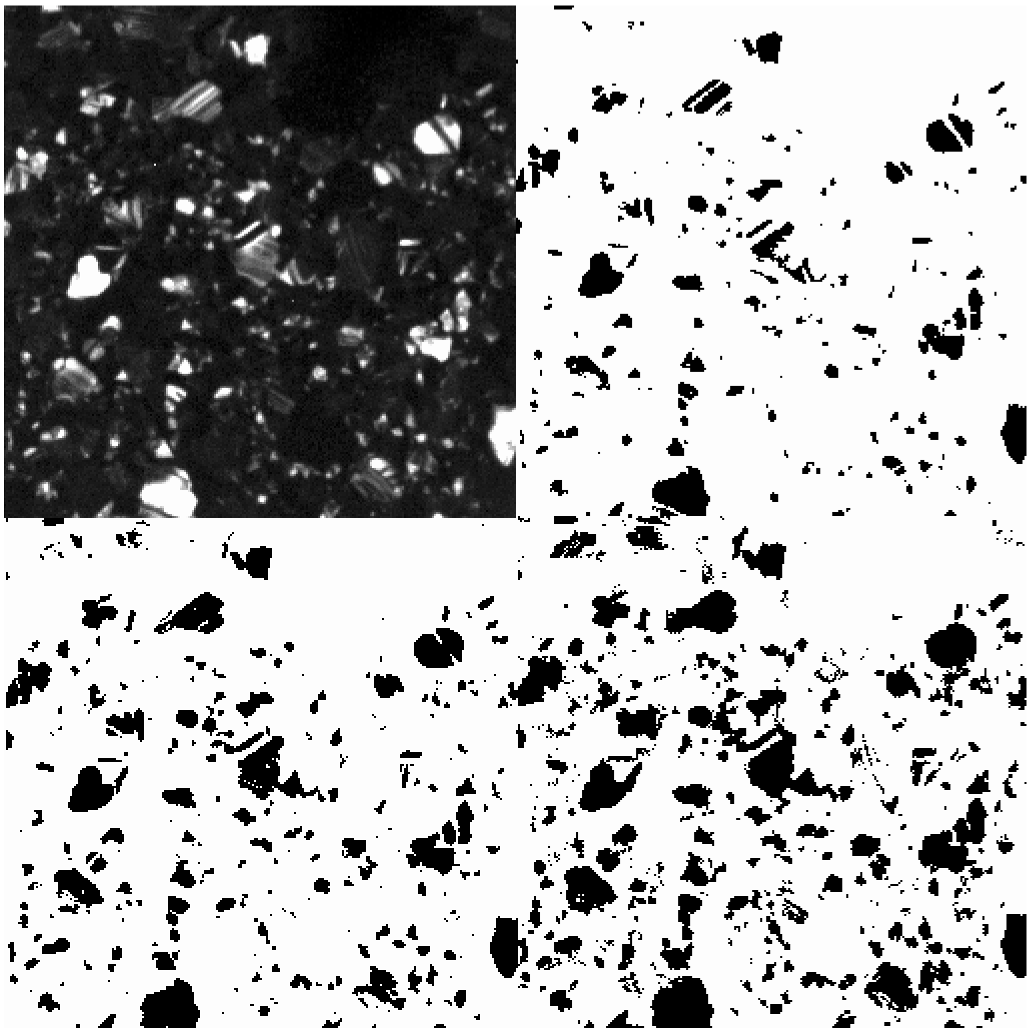
Thresh = 210	**220**	**111**	**200**	**311**
s = 2-3	1.614242	1.702474	1.654951	1.667205
s = 13-17	1.507534	1.705807	1.766551	1.754871
Thresh = 188	**220**	**111**	**200**	**311**
s = 2-3	1.608247	1.602977	1.582639	1.547907
s = 13-17	1.313757	1.511971	1.436904	1.525707
Thresh = 150	**220**	**111**	**200**	**311**
s = 2-3	1.568375	1.52711	1.48587	1.419023
s = 13-17	0.92678	1.110822	1.163722	1.046953



The images and graphs follow on the next pages. Note how by *threshold=188* that the 220 diffracting grains *log (N(s)) vs. log (1/s)* curves lay significantly beneath those of 111, 200, 311. This means that the total area of the 220 diffracting grains for all boxes is smaller than the others. If the film is 220 oriented, the number of grains capable of diffracting 220 should be reduced, and those capable of 111, 200, 311 reflections are increased. Ergo, this analysis supports 220 oriented Ag resulting from annealing at 300 ^{o}C of the as deposited film. Note also the rapid rise of the 311 *log (N(s)) vs. log (1/s)* curve with thresholding. Lastly, I thought the *threshold=210* was a pretty good binary representation of the overall shape of the diffracting grains. Conclusions follow pics.

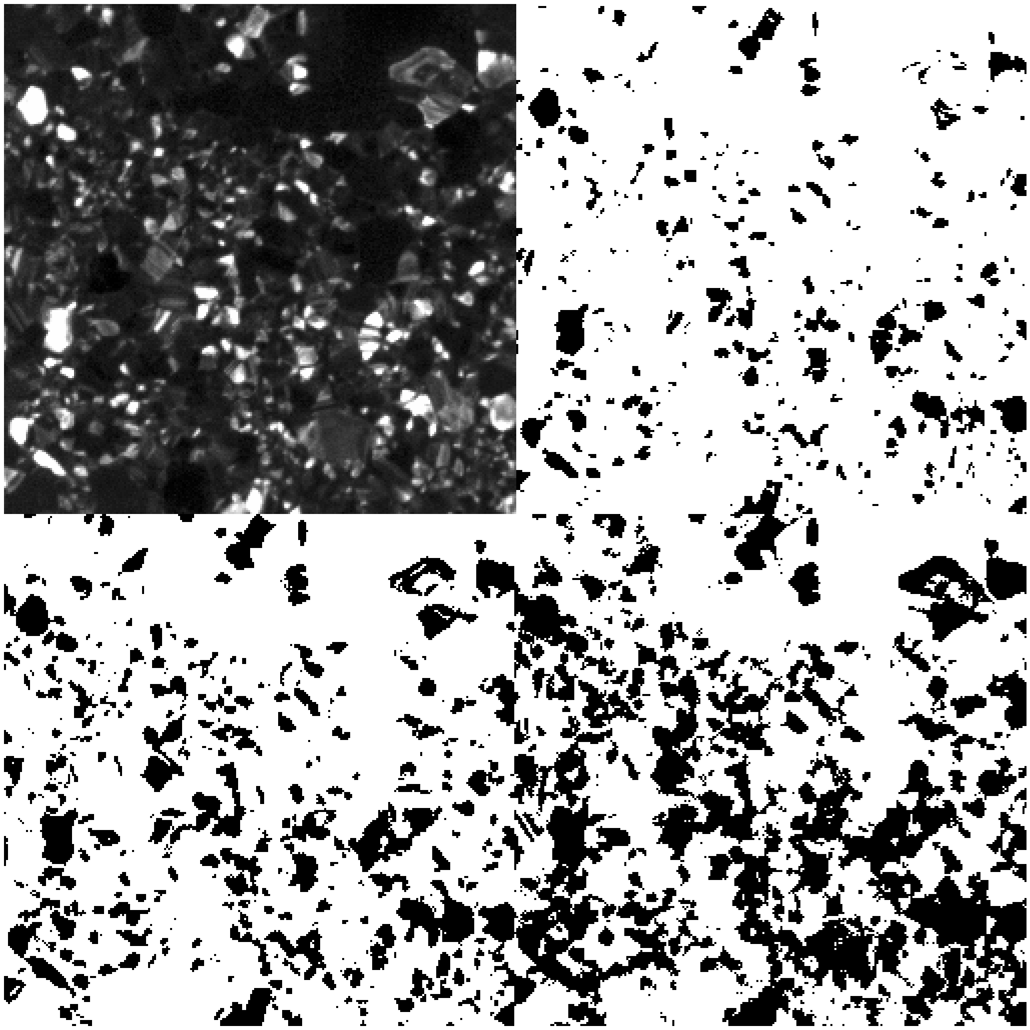




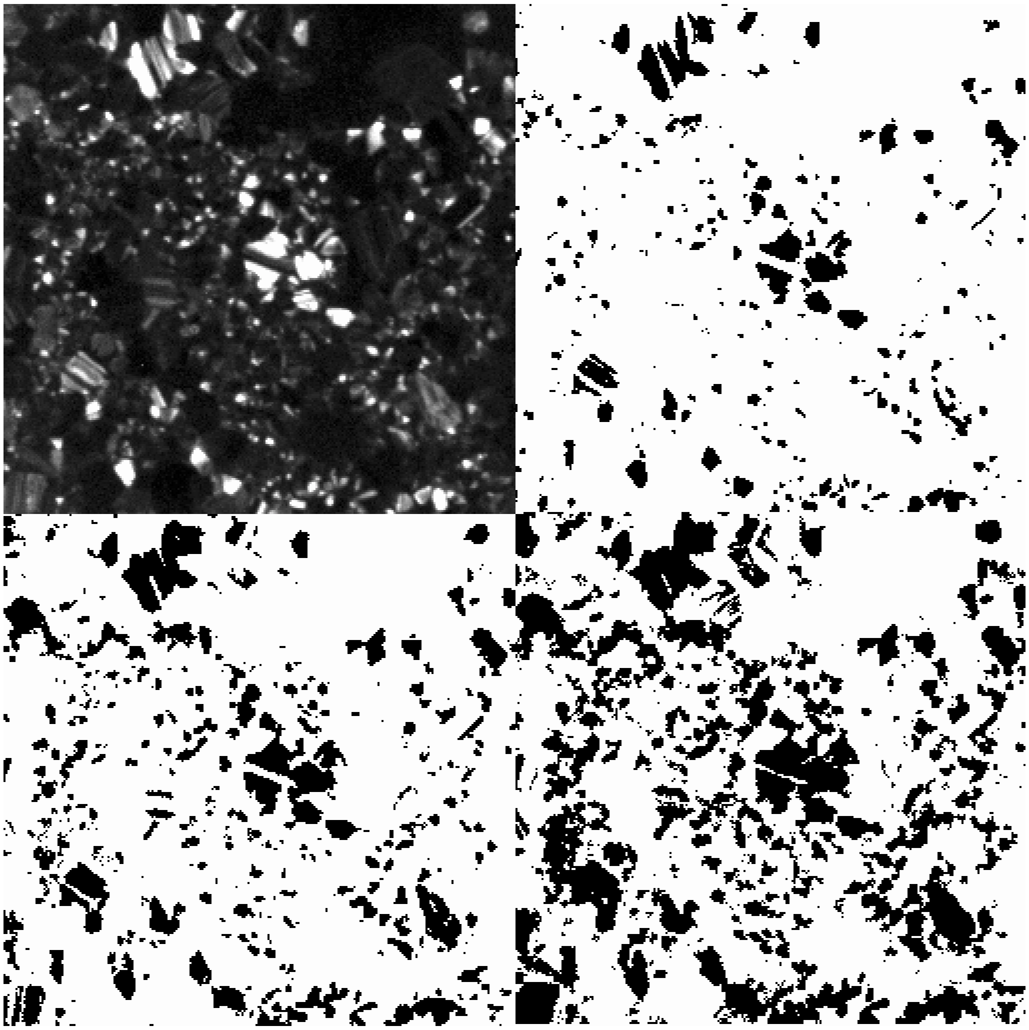


220 hollow cone and threshold=150, 188,210. See "handprint" 45 pixels wide.

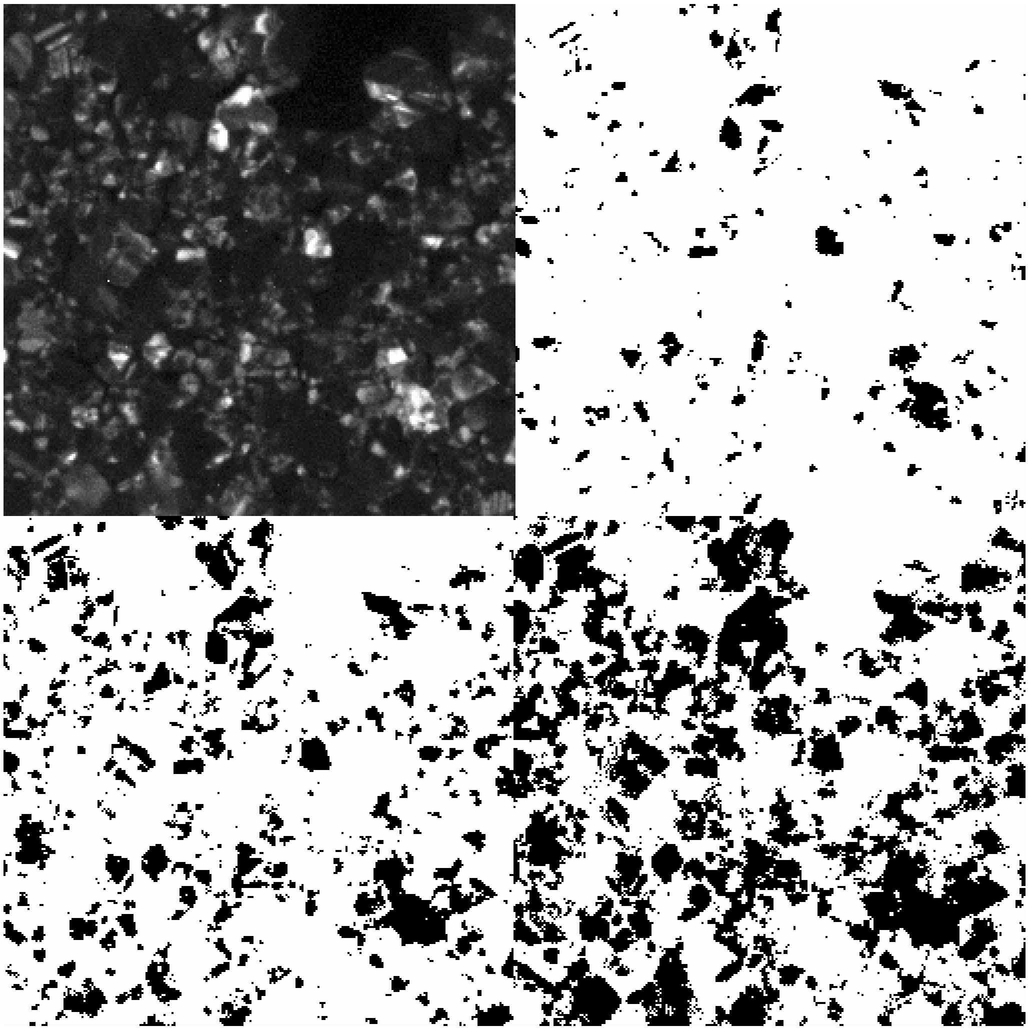
Compare the "dark" non-diffracting grains in the original image to the white areas of the binary ones. The shape of the dark areas is captured more accurately in the highest threshold setting.



111 hollow cone and threshold=150, 188, 210.



200 hollow cone and threshold=150, 188, 210.



311 hollow cone and threshold=150, 188, 210.

Conclusions.

- *D~1.6-1.7* for all the grains diffracting in the hollow cone darkfield images.
- The *threshold=210* setting may be just a bit shy of the optimal setting as there is still some structure in the 220 diffracting grains.
- Setting the threshold to achieve a fairly constant *D* in the range estimated from visual inspection of the image removes some of the operator feel from the process, and allows some data to be collected without manual circling of grains or outlining.
- The analysis supports a 220 oriented film.
- Analysis of a bi-crystal series of images using the same assumptions would be helpful. The assumptions that help determine the regime of *D* which may change with microstructure.
- Hollow cone and the fractal analysis may be helpful in imaging and gaining quantitative information about Ni_3P segregating in grain boundaries of plated Ni films on heating.

Lastly, I found a much more sophisticated program call HarFA (harmonic fractal analysis) which creates some beautiful data, is very fast, and allows a larger range of boxes to be analyzed. It calculates the fractal dimension spectrum as well, so you can see structure in the *D(s)* that corresponds to the particle size distribution and distribution of spacings between particle edges. I will provide the analysis of the hollow cone images with this program shortly.