STUDY OF THERMAL PROPERTIES OF POROUS MATERIALS

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Abstract

The paper examines thermal properties of materials. The transient pulse method was used for specific heat, thermal diffusivity and thermal conductivity determination. The evaluation was performed with the help of mathematical apparatus used for study of fractal structures properties. The results that were obtained are the same as the results obtained by classical methods.

Key words: fractal structure, specific heat, thermal diffusivity, thermal conductivity, transient pulse method

1 Introduction

The article deals with the use of new data evaluation method, which was described in [1]. The method results from generalized relations that were designed for study of physical properties of fractal structures [2], [3]. As it is shown these relations are in a good agreement with the equations used for the description of time responses of temperature for the pulse input of supplied heat [4], [5], [6]. Thermal parameters (specific heat, thermal diffusivity and thermal conductivity) calculated are corresponding for both methods.

2 Theory

The dependence of fractal structures' (characterized by the fractal dimension D in E-dimension space) temperature on the distance from heat source h_T and on the time t was determined in [1] using the theory of the space-time fractal field [2], [3]

$$T = \frac{Q}{c_\mathrm{p}\rho}\left(\frac{8\pi a_0 t}{D-E+2}\right)^{(D-E)/2}\left(1-\frac{h^2}{4a_0 t}\right)^{(D-E+2)/2}. \tag{1}$$

In this term Q is the heat supply from the heat source, c_p is the specific heat capacity at constant pressure (J.kg^{-1}.K^{-1}), ρ is the mass density (kg.m^{-3}) a_0 is the minimum value of the thermal diffusivity $a = 2a_0/(D-E+2)$ (m^2.s^{-1}) for fractal dimension equal to the topological dimension of the space ($D = E$).
If the heat diffuses by the significantly smaller speed ($h^2 << a_0 t$, small distances or long times) the terms in parenthesis can be considered as significant in the expansion of exponential function ($1 - x \approx \mathrm{e}^{-x}$) and thus we can write

$$T = \frac{Q}{c_\mathrm{p}\rho(4\pi a t)^{(E-D)/2}}\cdot\exp\left(-\frac{h^2}{4at}\right), \tag{2}$$

where Q is the total heat transferred to the body from the heat source with the thermal conductivity $\lambda = c_{\mathrm{p}}\rho a$. The relation (2) is applicable for fractal dimensions D = 0, 1, 2 and topological dimension E = 3 published in [4], [5],[6], see Fig. 1.

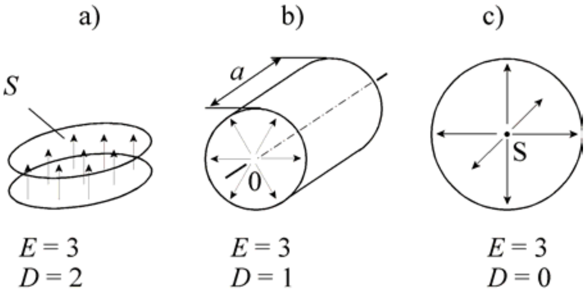

Fig. 1 Heat flow geometry for a) plane-parallel, b) cylindrical and c) spherical coordinates Euclidean space.

The maximum position can be determined by the derivation of (2) with the time

$$\frac{\partial \log T}{\partial \log t} = \left(\frac{D-E}{2} + \frac{h^2}{4at} \right) = 0 . \qquad (3)$$

From this equation the thermal diffusivity at the maximal time can be determined

$$a = \frac{h^2}{2t_{\mathrm{m}} f_{\mathrm{a}}} = \frac{h^2}{2(E-D)t_{\mathrm{m}}} , \qquad (4)$$

where f_a is a coefficient that characterizes the deformation of the thermal field [6]. This coefficient is equal to one for the ideal plane source $(E = 3, D = 2)$. The maximum temperature of the response for Dirac thermal pulse is obtained by introducing of the thermal diffusivity (4) in the term (2)

$$T_{\mathrm{m}} = \frac{Q}{c_{\mathrm{p}}\rho} \exp\left(\frac{D-E}{2} \right) \cdot \left(\frac{E-D}{2\pi h^2} \right)^{(E-D)/2} . \qquad (5)$$

From the ratio of equations (6) and (2) and with the use of the term (4)

$$\frac{T_{\mathrm{m}}}{T} = \left[\frac{t}{t_{\mathrm{m}}} \exp\left(\frac{t_{\mathrm{m}}}{t} - 1 \right) \right]^{\frac{E-D}{2}} , \quad \frac{T_{\mathrm{m}}}{T} = \left[\frac{t}{t_{\mathrm{m}}} \exp\left(\frac{t}{t_{\mathrm{m}}} - 1 \right) \right]^{\frac{E-D}{2}} \text{ respectively,} \qquad (6)$$

it is possible to definite the coefficient f_a (fractal dimension D respectively) for every point of the experimental dependence

$$f_a = E - D = \frac{2\ln(T_{\mathrm{m}}/T)}{\ln(t/t_{\mathrm{m}}) + (t_{\mathrm{m}}/t - 1)} , \quad f_a = E - D = \frac{2\ln(T_{\mathrm{m}}/T)}{\ln(t/t_{\mathrm{m}}) + (t/t_{\mathrm{m}} - 1)} \text{ respectively.} \qquad (7)$$

The relations on the left side are used for the temperature increase; the relations on the right side are used for the temperature decrease. The value of the coefficient f_a could be also affected by the geometry of sample [6] or by the finite pulse width, too [7].
From term (5) it is obtained the thermal capacity

$$c_{\mathrm{p}} = \frac{Q}{\rho T_{\mathrm{m}} h_T} \cdot \frac{f_{\mathrm{c}}}{\sqrt{2\pi \exp(1)}} = \frac{Q}{\rho T_{\mathrm{m}} h^{E-D}} \cdot \left(\frac{E-D}{2\pi \exp(1)} \right)^{(E-D)/2} \qquad (8)$$

and thermal conductivity of the studied fractal structure

$$\lambda = c_{\mathrm{p}}\rho a = \frac{Q}{2(E-D)T_{\mathrm{m}} t_{\mathrm{m}} h^{E-D-2}} \cdot \left(\frac{E-D}{2\pi \exp(1)} \right)^{(E-D)/2} , \qquad (9)$$

where f_a and f_c are the coefficients that characterize the deformation of the thermal field [6].

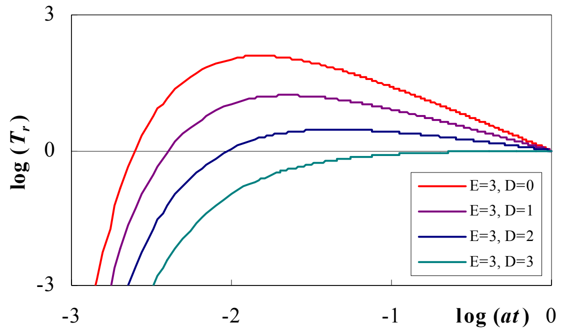

Fig. 2 Time dependency of the temperature response for the Dirac thermal pulse (for the heat flow geometry from Fig. 1 calculated by Eq. (2).

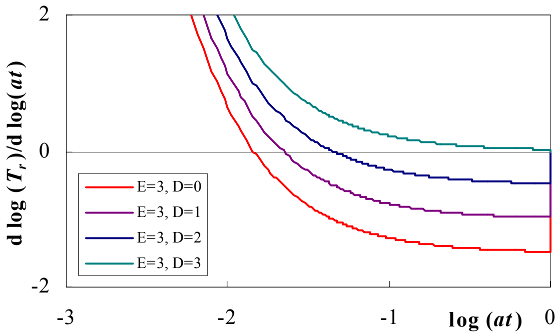

Fig. 3 Time dependency of the slope temperature response for the Dirac thermal pulse (for the heat flow geometry from Fig. 2 calculated by Eq. (3).

The Fig. 2 represents time-temperature dependencies (according equation (2)) calculated for spherical ($D = 0$), cylindrical ($D = 1$), planar ($D = 2$), and cubic ($D = 3$) geometry of the heat source (see Fig. 1). It is evident from the Fig. 2 and from the equation (3) that for $D = E$ the function meets maximum for the time $t \to \infty$.
All dependences for the long time intervals converge to the asymptote, which is longitudinal with the time scale. The intersection of this asymptote with the vertical scale determines the coefficient $f_a = (E - D)$ and thus the fractal dimension D that characterizes the specimen set-up (heat source, specimen, distribution of the temperature field, heat losses). When the value f_a is known it is feasible to determine the parameters of the studied thermal system with the aid of the (4) – (9) equations.

3 Experimental

The Thermophysical Transient Tester 1.02 was used for measuring of the responses to the pulse heat. It was developed at the Institute of Physics, Slovak Academy of Science [7]. The order of the experiment is described in [1].
Thermal responses from Slovak Academy were used for the data evaluation. The measured sample was round shaped with diameter $R = 0{,}03$ m. Its density was $\rho = 77{,}9$ kg.m^{-3} for its thickness $h = 0{,}0075$ mm, the thermal conductivity was $\lambda = 0{,}0254$ W.m^{-2}.K^{-1}.

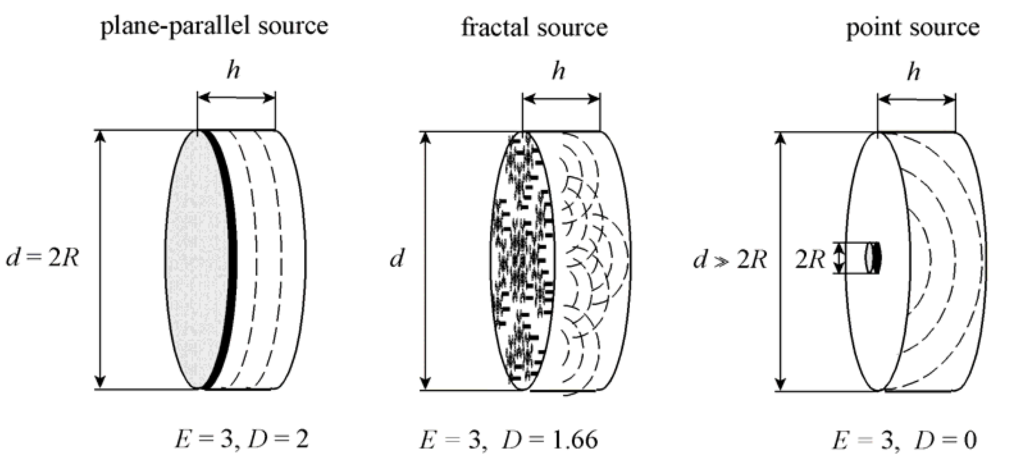

Fig. 4 Current flow geometry: a) plane-parallel, b) fractal, c) point (for different ratio of length contact respectively) source.

In Fig. 4 three possible configurations of experiment arrangements are shown. In Fig. 4a the diameter of specimen is equal to the diameter of a heat source, in Fig. 4c diameter of a heat source is far smaller than specimen's diameter. Fig. 4b shows the real situation, when the heat is delivered irregularly (either from the source of finite size (capacity) or from a source with specific composition of heat sources – e.g. hot-disc).

4 Results

The Fig. 5 represents the typical time responses of temperature for the step wise of input power. The coefficient f_a (fractal dimension D respectively) of the fractal heat source for every point of the experimental dependence (measured temperature depended on time) was calculated using the Eq. (7). The fractal heat source characterizes the distribution of the temperature in the specimen in specific time. From the Fig. 6 it is evident that for very short time there is the value of the fractal dimension $D \approx 2$ and therefore, the plane heat source is formed. The value of the fractal dimension decreases with increasing time value since the heat disperses into the space. From the time $\tau_1 \approx 16$ s (the intersection of tangents of the curves) the fractal dimension is getting settled to the value $D \approx 0{,}15$. The spatial distribution of the temperature in the sample does not change yet in this area. It is possible to determine the coefficient of the heat source $f_{a0} = 1$ and the diffusivity of the specimen $a \approx 4.679 \cdot 10^{-7}\,\mathrm{m^2 s^{-1}}$ from the extrapolated value of the fractal dimension to the time $t = 0$ s. This value is identical to value determined by the Institute of Physic, Slovak Academy of Sciences, Bratislava.
The deviations between the experimental (the black curve) and the model (the red curve) response obvious in the descending part of the characteristics are caused by the heat dissipation from the material via the cylinder surface of the specimen. This causes a faster decrease of the temperature than the theory predicted. The course of the temperature deviation between the model and experimental characteristic is illustrated as the blue curve in the Fig. 5. Negative values of functional dependence show the heat dissipations from the specimen. This deviation has also its own extreme (minimum), which means that for the value of time $t_m < 250\,\mathrm{s}$ heat losses are rising and for longer time intervals heat loses are smaller.

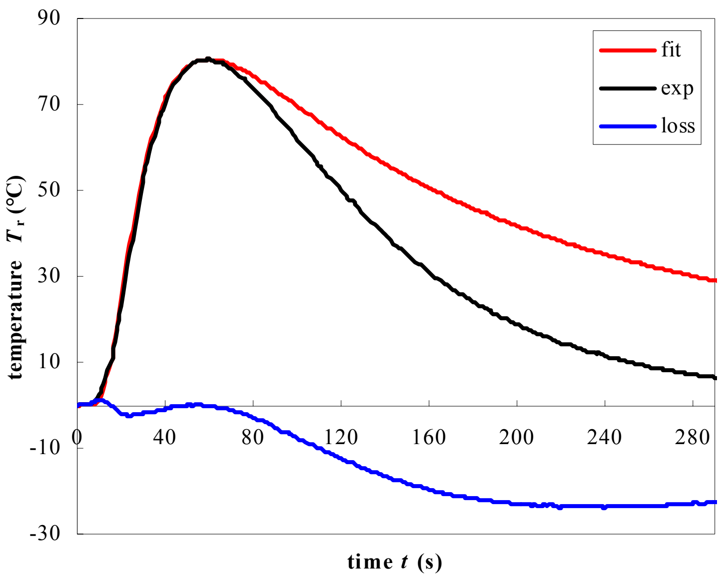

Fig. 5 Thermal response of the sample measured by the pulse transient method (black – experimental data, red – model data, blue – difference between experimental and model data).

From the descending characteristic we can again determine, by using (7) for each point of experimental dependence of measured temperature on time, coefficient f_a, fractal dimension D of the fractal source "*of cold*" presented by specimen surface. From Fig. 6b it is evident that there are not any cold spots over the surface of specimen for time intervals close to the maximum (the fractal dimension of heat spots is equal to the topological dimension $D \approx 3$. With rising time the value of fractal dimension of decreasing temperature is smaller again until the value $D \approx 2$. This is a fractal dimension of the specimen surface. The time constant of this descent is $\tau_2 \approx 86$ s. From the proportion of time constants (expecting that diffusivity of the material does not change) we can presume the distance between the source of heat dissipation and the thermocouple $x = h\sqrt{\tau_2/\tau_1} \approx 0.017\mathrm{m}$. This value approximately responds to the specimens' radius.

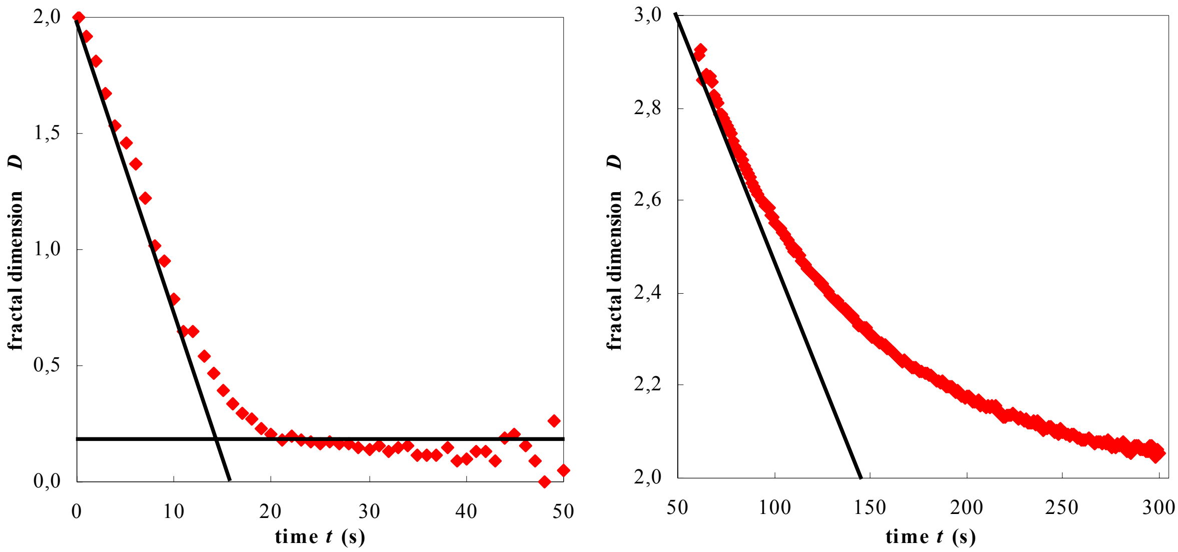

Fig. 6 Fractal dimension of the heat distribution in the specimen from a) increased and from b) decreased part of characteristics.

5 Conclusion

In this article, the results of thermal responses to the pulse of supplied heat evaluations are discussed. To interpret the outcomes, the simplified heat conductivity model is used [1]. The model is based on expectations published in [4]. Results show the image of heat distribution in the specimen, in various time intervals after the heat supply from the source. These evaluations could be used for more accurate determination of the thermal parameters of studied matters.

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