FRACTAL PHYSICS

O. ZMESKAL, M. BUCHNICEK and P. BEDNAR
Brno University of Technology, Faculty of Chemistry,
Institute of Physical and Applied Chemistry,Purkynova 118, 612 00 Brno, Czech Republic
E-mail: zmeskal@fch.vutbr.cz

Fractal geometry has been widely used nowadays in many scientific areas. Fractals seem to be very powerful in describing natural objects on all scales. Fractal dimension and fractal measure are the crucial parameters for such description. The authors are trying to link these quantities to a description of fundamental physical laws, stressing their fractal basis. A complex description of fractal fields conservative forces in the Euclidean space is presented. The physical quantities used to describe different properties of physical reality (electrical, gravitational, thermal, acoustic, etc.) are defined. This imply that there are no different laws, which act on different scales but there is a small set of universal properties, which act in different dimensional spaces as in El Naschie's Cantorian infinite theory. The mathematics of fractal–Cantorian geometry is used to describe electrical field properties of systems with different charge distribution and thermal properties of bodies.

Keywords: physical field, fractal physics, fractal geometry, fractal structures

1 Introduction

Most of natural objects are, in itself, fractals [1]. These fractals can be described using fractal measure (K) and fractal dimension (D). Fractal measure defines the magnitude of the space coverage using the elementary cell (e.g. percentage coverage), while fractal dimension describes the trend of change in coverage as a function of measuring cell size (its radius can be bigger or smaller than radius of the elementary cell). The coverage of E - dimensional Euclidean space E_n ($E = n$) of fractal structure was defined as [2]

$$F(r) = \frac{N(r)}{r^E} = K \cdot r^{D-E} , \tag{1}$$

where $N(r)$ is the coverage of radius r by elementary quantity. Their relation to the density of this quantity $q_0(r)$(e.g. density of mass or electric charge)

$$q_0(r) = e\,F(r) = eKr^{D-E} , \tag{2}$$

where e is the size of the elementary quantity. We will calculate the physical quantity $Q_0(r)$ by equation

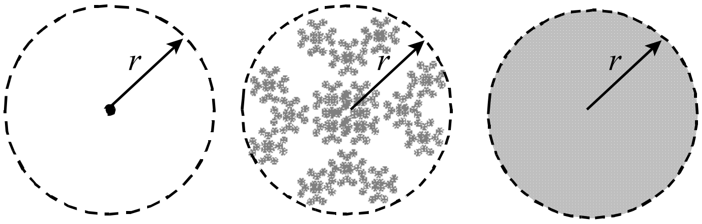
$$Q_0(r) = \int_{V*} q_0 \,\mathrm{d}V^* = eK \frac{E\,r^D}{D} = \frac{E\,r^E}{D} q_0(r), \tag{3}$$

where $\mathrm{d}V^* = \mathrm{d}(r^E)$ is an elementary volume of the E-dimensional space.
Fractal dimension can vary in the interval $D \in \langle 0, E \rangle$. In the limit cases

- The mean density of a quantity $q_0(r)$ for $D = 0$ (e.g. there is a constant number of objects independent of the space size, $Q_0(r) = eK$) decreases according to the equation $q_0(r) = eK \cdot r^{-E}$ (for $r > 1$).
- The coverage of space is independent of the space size for $D = E$ (e.g., the space is homogeneously filled) and is equal to the fractal measure $q_0(r) = eK$. The number of objects in the circumscribed area (for $r > 1$) increases with its size $Q_0(r) = eK \cdot r^E$.

The situation for 2D space ($E = 2$) is presented in Fig. 1; the different constants K and K^* are related to a radial demarcation of space (the square demarcation in the box counting was replaced with circle demarcation). The middle figure demonstrate a planar structure ($E = 2$) with fractal dimension $D = 5/3$.

$D = 0$ $E = 2$	$D = 1.66$ $E = 2$	$D = 2$ $E = 2$



$q_0(r) = eK\cdot r^{!2}$	$q_0(r) = eK\cdot r^{!0.34}$	$q_0(r) = eK$
$Q_0(r) = eK^*$	$Q_0(r) = eK^* r^{1.66}$	$Q_0(r) = eK^*\cdot r^2$

Fig. 1. The coverage of area a) point character of quantity, b) fractal quantity, c) homogeneously filled area.

2 Physical field, potential and other quantities of fractal structures

From the density of quantity $q_0(r)$ given by (2) we can determine the radial field intensity E_r [3] and the corresponding potential V_r [4] of physical (e.g. thermal, electric or gravitational) field

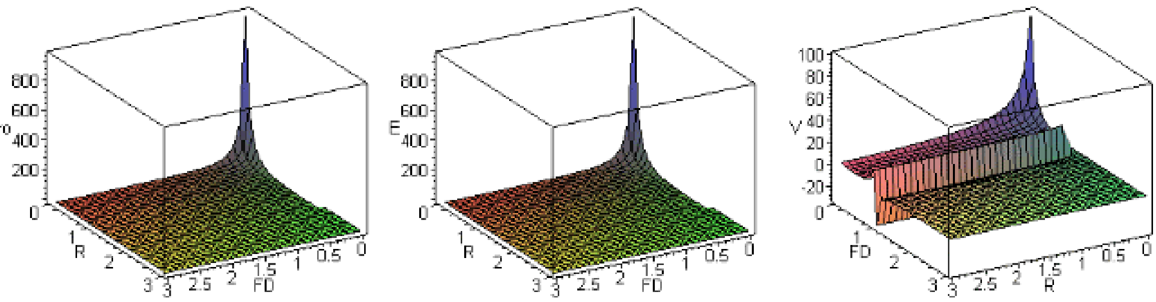
$$E_\mathrm{r} = k(eK)\cdot\frac{r^{D-E+1}}{D}, \qquad V_\mathrm{r} = -k(eK)\cdot\frac{r^{D-E+2}}{D(D-E+2)}. \tag{4}$$

From the density of quantity (4) and from the potential of physical field (4) the density of energy

$$w = \rho V_\mathrm{r} = -\frac{D(D-E+2)}{k}\cdot\left(\frac{V_\mathrm{r}}{r}\right)^2 \tag{5}$$

can be also determined.

Fig. 2 shows the dependences of quantity $q_0(r)$ density, the radial field intensity E_r and the corresponding potential V_r of physical field on radius r and the fractal dimension D. A discontinuity, which separate two areas with different physical properties (as it will become evident in following part), is evident for fractal dimension $D = 2$ in Fig. 2c).



a) b) c)

Fig. 2 The dependences of physical quantities on the distance r and the fractal dimension D a) radial density of physical quantity $q_0(r, D)$, b) radial part of the intensity of field $E_r(r, D)$ and c) radial part of the potential of the physical field $V_r(r, D)$ for topological dimension $E = 3$

3 Application to real physical fields

The density of temperature radiation energy is possible to describe, according to Eq. (5), as

$$w_{\mathrm{T}}(r) = k\,n(r)\,T_{\mathrm{r}}(r) = -\hbar c\,\frac{K^2 r^{2(D-E+1)}}{D(D-E+2)}\,, \tag{6}$$

respectively

$$w(T_{\mathrm{r}}) = K\cdot\left[\frac{D(E-D-2)}{K\hbar c}\right]^{\frac{E-D}{E-D-2}}\cdot\left(kT_{\mathrm{r}}\right)^{\frac{2(E-D-1)}{E-D-2}} = K\,kT_r\left[\frac{kT_{\mathrm{r}}\,D(E-D-2)}{K\hbar c}\right]^{\frac{E-D}{E-D-2}} \tag{7}$$

where $T_{\mathrm{r}}(r)$ is the temperature, $k = 1.380658\cdot10^{-23}\,\mathrm{J.K^{-1}}$ is the Boltzmann constant, $\hbar = 1.05457266\cdot10^{-34}\,\mathrm{J.s}$ is the modified Planck constant and $c = 2.99792458\cdot10^{8}\,\mathrm{m.s^{-1}}$ is the speed of light in vacuum.

In the same way, with the help of Poisson equation $\Delta\varphi_{\mathrm{r}} = -e\,n(r)/\varepsilon_0$ we can also express the density of electric (electromagnetic) field energy [4]

$$w_{\mathrm{EM}}(r) = e\,n(r)\,\varphi_{\mathrm{r}}(r) = -\frac{e^2K^2}{\varepsilon_0}\,\frac{r^{2(D-E+1)}}{D(D-E+2)}\,, \tag{8}$$

where $\varphi_{\mathrm{r}}(r)$ is the gravitational potential, $e = 1.60217733\cdot10^{-29}\,\mathrm{C}$ is the elementary charge and $\varepsilon_0 = 8.854187817\cdot10^{-12}\,\mathrm{F.m^{-1}}$ is the permitivity of vacuum and the density of gravitational energy reeds [4]

$$w_{\mathrm{G}}(r) = m_{\mathrm{p}}\,n(r)\,V_{\mathrm{rG}}(r) = -G_{\mathrm{N}}m_{\mathrm{p}}^2K^2\,\frac{r^{2(D-E+1)}}{D(D-E+2)}\,, \tag{9}$$

where $V_{\mathrm{rG}}(r)$ is the gravitational potential, m_{p} is the elementary mass (e.g. mass of the proton) and $G_{\mathrm{N}} = 6.67259\cdot10^{-11}\,\mathrm{m^3.kg^{-1}.s^{-2}}$ is the Newtonian constant of gravitation.

If we presume that fractal parameters (fractal dimension D and fractal measure K) of an analysed structure are the same a ratio between density of electric (electromagnetic) field energy and density of temperature radiation energy can be easily expressed [5]

$$\frac{w_{\mathrm{EM}}}{w_{\mathrm{T}}}=\frac{e^2}{\hbar c\varepsilon_0}=\frac{4\pi}{\overline{\alpha}_{\mathrm{EM}}}, \qquad \frac{w_{\mathrm{G}}}{w_{\mathrm{T}}}=\frac{G_{\mathrm{N}}m_{\mathrm{p}}^2}{\hbar c}=\frac{1}{\overline{\alpha}_{\mathrm{G}}}, \tag{10}$$

where $\overline{\alpha}_{\mathrm{EM}}$ is the electromagnetic inverse coupling constant and $\overline{\alpha}_{\mathrm{G}}$ is the gravitational inverse coupling constant respectively. These parameters (10) thus represent the aliquot part of energy density of their individual forms considering the state when $\overline{\alpha}=1$. This means that interaction between objects is represented equally by physical particles and photons. These constants are independent on the size of the analysed space in homogeneous isotropic environment (in the same manner as fractal measure K and fractal dimension D).

4 Conclusion

The contribution adopted fractal geometry mathematic apparatus to describe properties of the gravitational, electrical and temperature fields. Fundamental quantities of these fields (intensity, potential and density of energy) and their nexus were defined and discussed. The conclusions are in accordance with the El Nashie's golden mean field theory [6], [7].

References

[1] Mandelbrot BB. Fractal geometry of nature. New York: W.H. Freeman and Co.; 1983

[2] Barnsley MF. Fractals everywhere. London: Academic Press, 1993

[3] Zmeskal O, Nezadal M, Buchnicek M. Fractal–Cantorian Geometry, Hausdorff Dimension and the Fundamental Laws of Physics. Chaos, Solitons & Fractals 2003; 17: 113–119

[4] Zmeskal O, Nezadal M, Buchnicek M. Field and potential of fractal–Cantorian structures and El Naschie's $\mathcal{E}(\infty)$ theory. Chaos, Solitons & Fractals 2004; 19: 1013–1022

[5] Zmeskal O, Buchnicek M, Bednar P. Coupling constants in fractal and cantorian physics. Chaos, Solitons & Fractals 2004; 22: 985–997

[6] El Nashie MS. On a class of general theories for high-energy particle physics. Chaos, Solitons & Fractals 2002; 14: 649–668

[7] El Nashie MS. A review of E infinity theory and mass spectrum of high energy particle physics. Chaos, Solitons & Fractals 2004; 19: 200–236