FRACTAL ANALYSIS OF PRINTED STRUCTURE IMAGES

Zmeškal Oldřich, Nežádal Martin, Komendová Barbora, Julínek Martin, Bžatek Tomáš

Institute of Physical and Applied Chemistry, Brno University of Technology, Brno, Czech Republic

ABSTRACT

Imaging photometry, the new modern method of evaluation of the quality of print is discussed in the following article.

Authors explain preparation of reproducible image structures; correction of the images (e.g. thermal noise, non-homogeneity of illumination, gamma correction) and main attention is aimed to the evaluation of quality of prints (different grades of paper) and the evaluation of printed areas with regard to used printing technology.

1. INTRODUCTION

Traditional methods used in polygraphic industry utilize special devices like *densitometer* or *colorimeter* to determine quality of the print. *Imaging photometer* is another special device used more often nowadays for digitization of printed images. The device uses scanner, digital camera or camcorder for digitization of prints. All of these devices can be used also separately for measurement of the quality of final prints as well as for analysis during print process. Their use also allow for increased automation of process and faster analysis. Problems with output of printers – decreased quality of prints - can be then recognized immediately and machine adjusted to fix the error. Another advantage is possibility of recording sample images for detail post mortem analysis. Following types of special analysis can be performed to monitor quality of the process:

- Analysis of quality of the prints.
- Analysis of quality of the print points reproduction.
- Analysis of print material homogeneity.
- Analysis if printed colour homogeneity on the surface of the print material.
- Analysis of quality of the print edges and sharpness of details.
- Analysis of exactness of colour of the print.

Following paragraphs tries to examine and explain some of the methods used to perform analysis listed above.

2. IMAGE ANALYSIS METHODS

Source images for analysis are recorded exclusively using recorders (camera, camcorder) utilizing CCD technology. Although CCD technology has to be presented it is not the only technology used for recording images. It is often beneficial for the quality of recorded images to use appropriate combination of devices with CCD based recorder as a final element (e.g. combination of microscope – digital camera, as shown on *Figure 1*). Quality of recorded image depends on resolution of the recorder (directly related to number of pixels of CCD element) and colour depth (based on

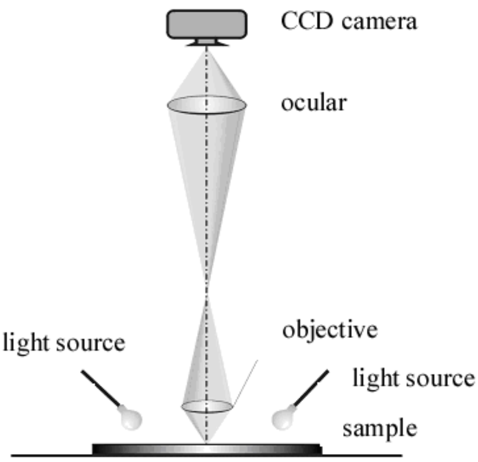

Figure 1 Recording system

quantization of D/A converter). The quality (resolution, colour depth) of recorders currently available is sufficient for analysis requirements. On the other hand there is also potential for introducing errors during creation of digital representation of images. Such errors have to be corrected later on to make images and results of analysis useful. The most often occurring errors are inhomogeneous image light, nonlinear brightness transformation (gamma correction) and thermal noise.

Image data are usually transformed prior to analysis itself (e.g. change of colour schema – RGB, HSB, HLS, Gray Scale, image pixel data correlation (resizing, increasing or decreasing resolution, simple filtration of data).

After initial transformations, the analysis itself is performed. The analysis is detail characterization of printed areas. The analysis is not performed on single pixels but data are analyzed as a whole. The analysis algorithm makes use of linear (integral) data transformations. The transformations are performed multiple times with different base data so different properties of single pixels (based on whole image) are examined. In case of orthogonal transformations it is possible to obtain original data by linear (integral) transformation of inverse base data. Based is often formed by *discreet periodical functions* (e.g. 2D harmonic analysis) or *discreet space limited functions* (so called wavelets).

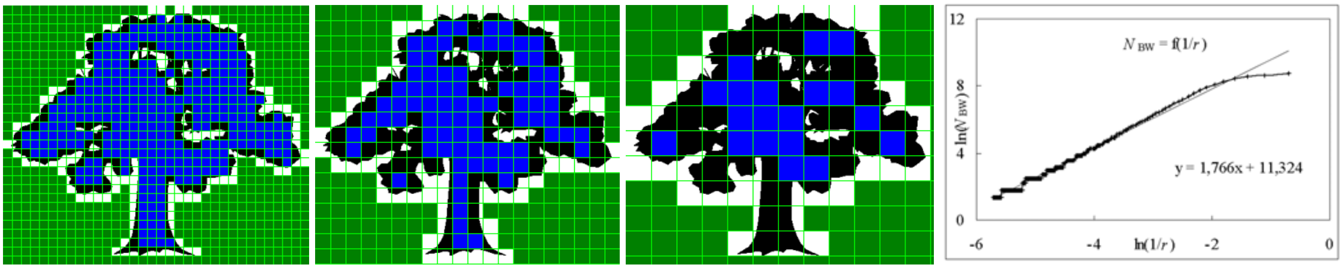

Figure 2 Box counting method

It is often beneficial to perform wavelet transformation during fractal analysis by box counting method, where squares of different sizes covering the image are counted (*Figure 2*). Function of black N_B, white N_W and black & white N_{BW} squares on their size ε can be used to determine three fractal dimensions; black and white area and their edge (D_{BBW}, D_{WBW}, D_{BW}). The dimensions D_{BBW} and D_{WBW} are then calculated as sum of black $N_{BBW} = N_B + N_{BW}$ (respectively white $N_{WBW} = N_W + N_{BW}$) and black & white squares. This can be expressed by equation (1) where K is so called fractal measure and D is fractal dimension (without indexes).

$$N(\varepsilon) = K \cdot \varepsilon^{-D} \qquad (1)$$

The equation shows us that fractal dimension of image structures can exist in interval $D \in \langle 0, 2 \rangle$. Fractal dimension has value $D = 0$, when and only when dimension colour is not presented in the image. The equation results in value of the fractal dimension $D = 2$ only when dimension colour is the only colour on the image. The edge of black and white will have fractal dimension $D_{BW} = 2$, when every single pixel interfaces with image of different colour. The example of the ideal situation when $D_{BW} = 2$ is Pean's curve (*Figure 3).* The curve fills whole image.

The fractal measure can exist in interval $K \in \langle 0, K_{max} \rangle$, where K_{max} is total count of image pixels. The value represents percentage of image filling by fractal structure (black, white or edge). For example when edge dimension $D_{BW} = 2$, the fractal measure will be equal of total image count, also $K_{BW}/K_{max} = 1$ (100 %).

It is of great importance to obtain and use *fractal spectrum* for analysis of colourful images. Fractal spectrum is function of fractal dimension and threshold value. Conversion of colourful images for analysis can be done using a lot of different techniques. Most common is conversion of colours to greyscale using

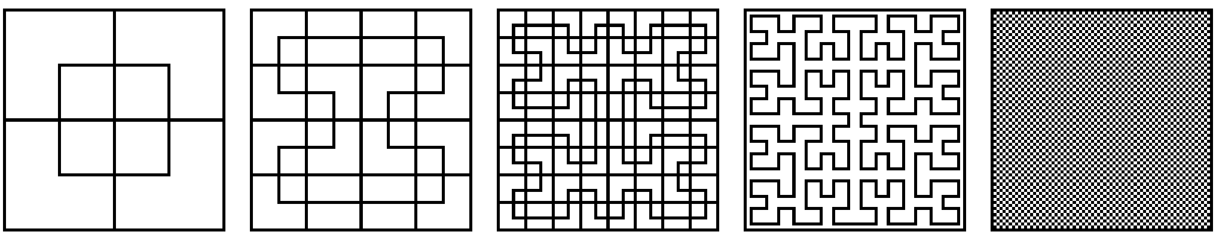

Figure 3 Pean's curve

equation $I = 0.299\,R + 0.587\,G + 0.114\,B$. All grey tones darker then chosen threshold are then treated as a black and all grey tones lighter then threshold value are treated as white pixels during analysis itself. The fractal spectrum is then obtained by calculating fractal dimension for threshold values of all grey tones (0 – 255). Sometimes it is also useful to treat as a black all pixels with colour in the range of standard deviation of threshold value and as a white all remaining pixels. It is also useful to analyze more then one property of colour in other cases (RGB, HLS, HSB).

Fractal spectrum can be used to obtain threshold value when fractal dimension and fractal measure reaches maximum values. Spectrum can be then used to judge value of homogeneity and area coverage of the prints.

where x is sum of analyzed points ($x = 100$) and r_S (or r_L) is radius determined by use of fractal analysis.

$$r_S = \sqrt{\frac{\varepsilon^2 N_{BBW}}{x\pi}}\ ,\ r_L = \frac{\varepsilon N_{BW}}{2x\pi}\,. \tag{4}$$

Difference between radiuses is function of magnitude of the net used for analysis as shown on *Figure 4d*)

$$\frac{r_L}{r_S} = \frac{N_{BW}}{2\sqrt{\pi\, x\, N_{BBW}}} \tag{5}$$

The radiuses are equal in ideal situation (the points are exact circles).

The deviation between calculated radiuses gets smaller with increased size of the net as shown on *Figure 4d*. This is mainly due ability of more exact determination of size of the area of the point and its edge. The remaining deviation is caused by differences of the

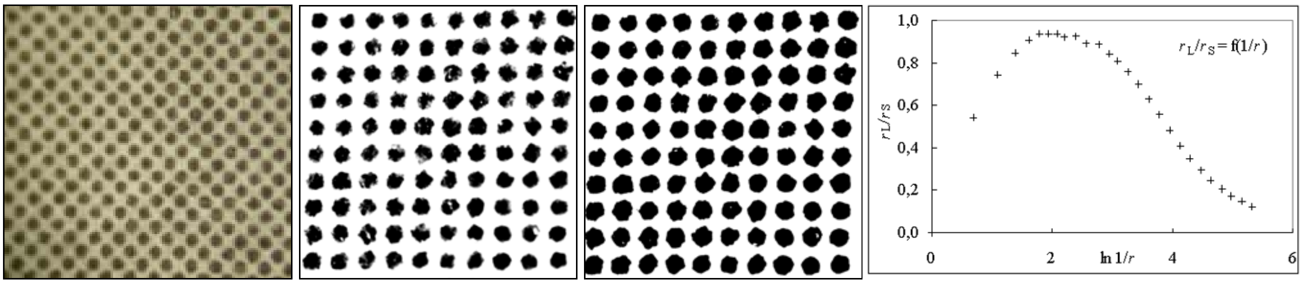

Figure 4 Real print points (a) cut to 100 and magnified to radius 35 in greyscale (b) b&w with threshold value of 135 (c) function of radius ratio r_L/r_S and magnification (d)

3. PRACTICAL USE OF FRACTAL ANALYSIS

Determination of quality of the print points

To determine quality of the print point, a property like deviation from ideal shape is tested. The ideal shape of the point may be elliptical, circle or square shape.

The base for analysis in this case is structure of print points of the same size and of same colour (same colour tone). Based on knowledge of amount of points in analyzed image (e.g. 100 on *Figure 4*) and using fractal analysis the average radius of points and deviation from circle like shape can be determined. The radius is calculated as a function of black colour covered area on the image N_{BBW} or as a function of black/white edge N_{BW} using equations

$$S = \varepsilon^2 N_{BBW} = x\pi\, r_S^2\,, \tag{2}$$

$$L = \varepsilon\, N_{BW} = 2x\pi\, r_L\,, \tag{3}$$

point's shape from circle (e.g. elliptical, square shape) and by not 100% coverage of the point by print colour. The deviation caused by shape difference can by correct by change of the coefficient. The general equation will then look like

$$\frac{r_L}{r_S} = \frac{N_{BW}}{2\sqrt{n\,\mathrm{tg}(\pi/n)x\,N_{BBW}}}\,, \tag{6}$$

where shape correction coefficient is $\sqrt{\pi/[n\,\mathrm{tg}(\pi/n)]}$.

Then for example for elliptical print point the eccentricity of the point can be defined in equation

$$S = \varepsilon^2 N_{BBW} = \pi\, abx = \pi\, a^2 x\sqrt{1-e^2}\ , \tag{7}$$

$$L \cong \varepsilon\, N_{BW} = 2\pi\, ax\left[1 - \frac{e^2}{4} - \frac{3\cdot e^4}{64} - \cdots\right], \tag{8}$$

where eccentricity itself is $e = \sqrt{1 - b^2/a^2}$, $(a > b)$.

There are two new variables in the equation that represents length of axes (a, b) of the ellipse.

The analysis of structure on *Figure 4* shows us that deviation of points shape from circle is $1 - r_L/r_S \approx 0.083$ (8.3 %), fractal dimension of the edge is $D_{BW} = 1.321$ and coverage

$$\frac{S}{S_{\max}} = \frac{K_{BBW} - K_{BW}/2}{K_{BBW} + K_{WBW} - K_{BW}} \approx 0{,}4158\,, \qquad (9)$$

i.e. 41.58 %. Deviation of results from initial parameters (for 40 % screen) is, as shown in equation (9) minimal.

Determination of print's homogeneity

To determine quality of laser print, the prints homogeneity is often required. The homogeneity is calculated from the fully covered areas of the prints (*Figure 5a, b*). The fractal spectrums are used to calculate homogeneity as a function of fractal dimension and threshold value *I*, used to determine black (0) and white (255) colour of the points, as shown on *Figure 5c*. The blank paper fractal spectrum (+) is very sharp as shown on *5d*. The paper's single grey tone value can be assigned ($I = 172$), with fractal dimension of value ($D = 1{,}910$) and coverage calculated as

$$\frac{S}{S_{\max}} = \frac{K_{BW}}{K_{BBW} + K_{WBW} - K_{BW}} \approx 0{,}8423\,, \qquad (10)$$

i.e. 84.23 %. The extreme on the right side of curve is cased by non-homogeneity of the paper.

A fractal spectrum of printed area (×) has maximum value near ideally black edge interface ($I = 28$). Fractal dimension of such spot is of $D = 1.881$ and area coverage is 72.6 %. From graph curve it is also clear that fractal dimension value decreases linearly with tone of the colour. The end of the curve shows background noise cased by non-homogeneities in the paper.

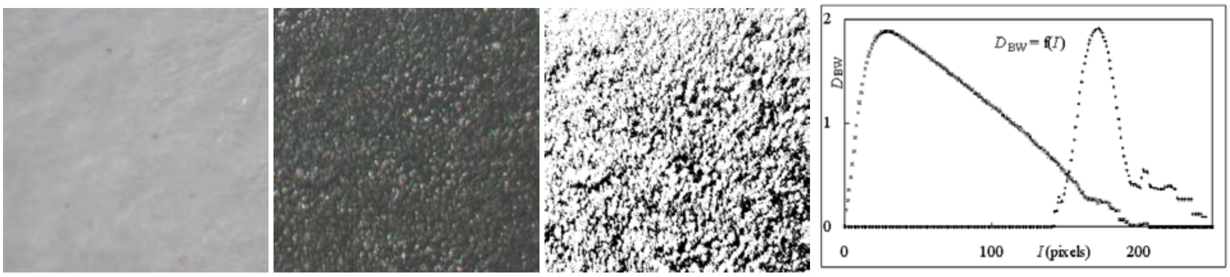

Figure 5 Homogeneity of the xerographic paper (a), laser print on xerographic paper (b) resample on threshold value of *I* = 28 (c), fractal spectra of the xerographic paper (+) and of laser print itself (×) (d)

Determination of quality of print point edges

Quality of the print edges is used for example to evaluate and divide original prints from its copies (*Figure 6a*). The evaluation is done by comparison of fractal spectra (*Figure 6c*) and shows us that images differ in both greyscale tone intensity and print paper homogeneity. The right side of max shows that original ($I_o = 167$) is darker then copy ($I_p = 180$). Left side max shows print quality of lines on used paper. The original paper is lighter ($I_o = 69$) then its copy ($I_p = 57$). The distance between max values shows that difference in spectra max distances is much higher for original print ($\Delta I_o = 98$) then for its copy ($\Delta I_p = 123$). Characteristic values for print technology, obtained from comparison of fractal dimensions of the original and its copy shows that print technology to make copy was most likely same as technology used for original ($D_o = 1.701$ a $D_p = 1.723$). The fractal dimension also shows that end of print lines are blurred (ideal line would have fractal dimension $D = 1$). The fractal dimension of printed area of the lines shows on the other hand that the covered area is much smaller ($D_o = 1.571$ a $D_p = 1.583$) then in ideal situation ($D = 2$).

The results of the area coverage and lines coverage are in correlation with values shown above. The fractal measure shows the area coverage (10) for the original 74.83 %, for the copy 76.26 % and coverage of printed area (9) for original 42.12 % and its copy 42.28 %.

Evaluation of colour reproduction

Fractal spectra displayed on right side of *Figure 7* are spectra of model colour print from left-hand side (with top square cyan and bottom yellow). The curve

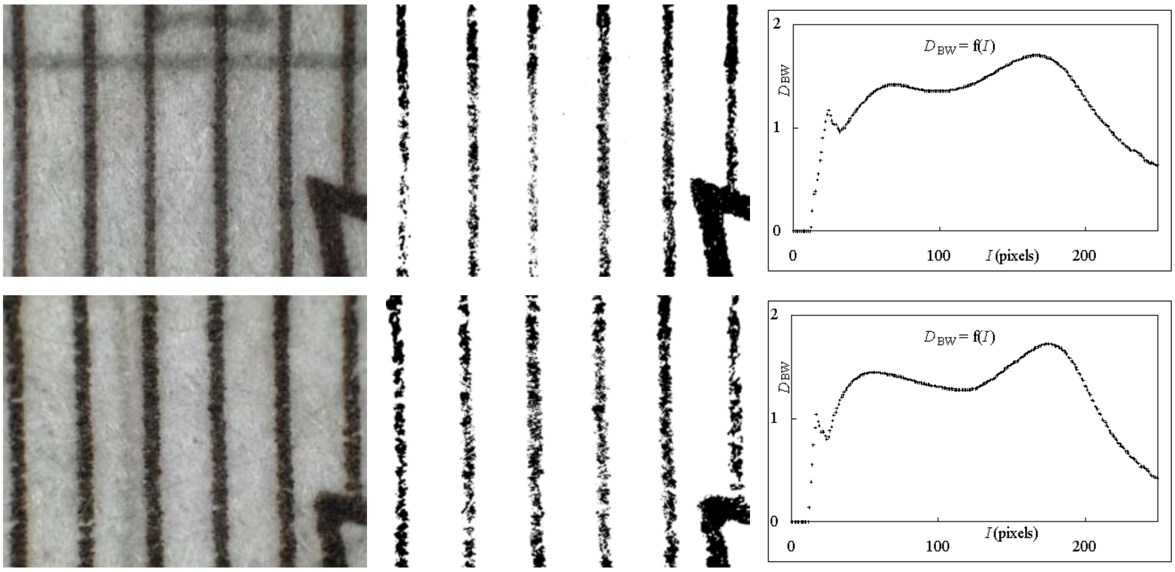

Figure 6 Reproduction of point edges of original (upstairs) a falsification (downstairs) of stamp microphotography (a), threshold (b) on position of first maximum of fractal spectrum (c). Values of threshold was taken for original $I_o = 69$, and for falsification $I_p = 57$.

shows couple of important peaks. The peaks from left to right green – composed from cyan and yellow, cyan, yellow and white – defining paper properties.

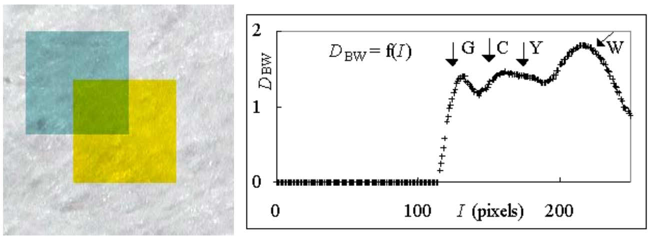

Figure 7 Model colour table and its print spectra

Summary

The article examines possibilities of use of the fractal analysis for evaluation of the quality of printed material and printed areas. The main requirement for analysis is to obtain high quality digital images of prints prior to analysis. It is also of great importance to ensure minimal loading of digital records with some common recording errors. It has been confirmed that results of such analysis forms base for objective comparison of papers homogeneity as well as for comparison of print colour homogeneity for full area print. The other very important area of use of fractal analysis of printed structures is comparison of quality of reproduction of the print points. Last couple of paragraphs tries to briefly introduce use of fractal analysis for evaluation of the colour separation for example for offset print technology.

More detail information about the fractal analysis and authors research is available on

http://www.fch.vutbr.cz/lectures/imagesci.

Literature

MANDELBROT B.B., *Fractal geometry of nature*. New York: W.H. Freeman and Co., 1983

ZMEŠKAL O., NEŽÁDAL M., BUCHNÍČEK M.: Fractal–Cantorian Geometry, Hausdorff Dimension and the Fundamental Laws of Physics. *Chaos, Solitons & Fractals* 2003; 17: 113–119

ZMEŠKAL O., NEŽÁDAL M., SEDLÁK O.: The Usage of Fractal Analysis for Evaluating the Quality of Print, *IV. Polygraphic Conference*, University of Pardubice, September 12. - 13., 2001, p. 92 – 101

ZMEŠKAL O., NEŽÁDAL M., BUCHNÍČEK M., FEDÁK J.: Fractal Analysis of Printed Structures, conf. *Polygrafia Academica 2000*, CHTF STU Bratislava, September 7 - 8, 2000, p. 207-213